ANALYSIS & DESIGN OF SINGLE SWITCH & PUSH-PULL CLASS-E AMPLIFIER

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by
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June, 1996

Certificate

It is certified that the work contained in the thesis entitled **ANALYSIS & DESIGN OF SINGLE SWITCH & PUSH-PULL CLASS-E AMPLIFIER**, by Amar Nath Tiwari, has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

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June, 1996 AMAR NATH TIWARI

Dedicated To

My Wife GEETA

ABSTRACT

Class-E amplifier operations are based on zero voltage, zero current switch turn-on. Forced turn-off of the switch is required, but the build-up of voltage across the switch is delayed. At "Optimum Operation" and under damped "Suboptimum Operation", its efficiency is ideally 100%. Analysis of basic class-E amplifier have been done by (i)Using state equations, (ii)Assuming constant input current and sinusoidal output current & (iii)Assuming constant input current & non-sinusoidal output current. The second analysis gives approximate design values of the amplifier and the last analysis gives more accurate design of basic class-E amplifier. The push-pull class-E amplifier have been analyzed by two methods, (i)Using state equations and (ii)Assuming constant input current and sinusoidal output current. The design equations obtained by the second method of analysis gives sufficiently accurate design values.

Contents

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List of Symbols

L_1, L_2	R F Choke inductors
C_1, C_2	Shunt capacitors
S_1, S_2	Switches
D_1, D_2	Diodes
L_o & C_O	Series inductor & series capacitor respectively
R &I_o	Equivalent load resistance & Output current respectively
V_s & V_{co}	Supply voltage source & voltage across series capacitor respectively
V_{c1}, V_{c2}	Voltage across shunt capacitors
I_1, I_2	Input currents
I_{sw1}, I_{sw2}	Current through switches
I_{c1}, I_{c2}	Current through shunt capacitors
I_{d1}, I_{d2}	Current through diodes
I_{s1}, I_{s2}	Sum of currents flowing through diode, switch and shunt capacitor
f & w	Frequency of operation in radian & Hz respectively
P_o	Output power to load resistance
P_i	Input power to the amplifier
I_{om}	Peak output current for fundamental component
I_{om_n}	Peak output current for n_{th} harmonic component
V_x	Voltage across equivalent reactance X($=wL_o - \frac{1}{wC_o}$)
V_{c1av}	Average voltage across shunt capacitor C_1
$V_{c1fundamental}$	Fundamental frequency component of C_1
ϕ	Phase angle of output current
ψ	Phase angle of series load impedance Z_o at operating frequency
η	Efficiency of the amplifier
I_{s1pk}, I_{s2pk}	Peak switch currents
V_{c1max}, V_{c2max}	Peak switch voltages
θ_{vmax}	Angle at which switch voltage is maximum
R_{dc1}, R_{dc2}, R_{dc}	Equivalent load offered by the amplifier on its supply voltage
D & Q_l	Duty-cycle & Quality factor respectively

Chapter 1

Introduction

Class-E tuned amplifiers are high efficiency amplifiers. The high efficiency is achieved by reducing collector power dissipation of switching transistor used in the amplifier. The reduction in switching loss, reduces the heat sink volume and weight considerably or reduces the junction temperature rise, thereby decreases the transistor failure rate.

At optimum operation of class-E amplifier, its efficiency is ideally 100%. The switching transition "off to on" is with the condition that collector-to-emmiter voltage and slope of the voltage waveform must be zero. "On to off" transition is with delayed voltage rise across the switch. Hence, for optimum operation, ideally there are no switching losses. If the switching "off to on" transition time is significantly large, the power dissipation will be minimum because of zero slope of voltage across the switch. The output power, normalized to the product of switch peak voltage and peak current will be maximum at a switch duty-cycle 50%. In the thesis, the design of basic class-E amplifier and push-pull class-E amplifier have been done with 50% duty-cycle of operation. Basic class-E amplifier has very simple configuration, because of single switch. This amplifier output response has harmonics in which first two harmonics are significant. It requires harmonic filter for radio-frequency applications. The push-pull configuration of this amplifier have output response with no even harmonic components.

Organization of Thesis

Chapter-2 of the thesis is mainly dedicated to analysis and design of basic class-E amplifier. Section 2.1 describes the basic criteria imposed on switch voltage, circuit configuration of basic class-E amplifier and its operation, other possible circuit configurations of class-E amplifier and different application of class-E amplifier. Section 2.2 is describes exact analysis of basic

class-E amplifier using state equations. State equations of the amplifier for two modes are written and method of solution has been described. In section 2.3 based on solutions of exact analysis of section 2.2, simulation studies have been done. Curves describing for transient behaviour and steady-state behaviour of the amplifier response were plotted. Latter a study has been carried out on following:

- Effect of change in power with supply voltage.
- Effect of change in load resistance.
- Effect of simultaneous change in load resistance and series tuned inductance.
- Effect of change in power with frequency.

Section 2.4 gives harmonic analysis of load current and voltage across the switch. Section 2.5 describes analysis of the class-E amplifier assuming constant input current and non-sinusoidal output current. First the analysis has been done by assuming output current to be fundamental. Latter, finite number of harmonics have been included in the analysis. A comparison of the two analysis with the exact analysis (described in section 2.2) is given. Section 2.6 gives design equations, which are derived from the analysis described in section 2.5. Approximate design equations were given based on the analysis, assuming sinusoidal output current.

Modified procedure for more accurate design has been described, it is based on the analysis, assuming non-sinusoidal output current. The design method is for the following two options:

1. Constant P_o, Q_l and f.
2. Constant R, Q_l and f.

Chapter-3 deals with analysis and design of class-E push-pull amplifier. First section of the chapter introduces with the configuration and possible modes of operation of the class-E push-pull amplifier. Section 3.2 gives exact analysis of push-pull class-E amplifier with two modes of operation, using state equations. State equations for the two modes of the circuit operation were written and solved with a set of given circuit elements. Steady-state curves of the circuit parameters have been plotted. In section 3.3, harmonic analysis of the output load current were carried out for steady-state and optimum operation. Section 3.4 gives the analysis of push-pull class-E amplifier, assuming constant input current and sinusoidal output current with two modes of operation. The results of the analysis of section

3.4 and section 3.2 were compared. In section 3.5 design equations for optimum operation of push-pull class-E amplifier were derived from the analysis given in section 3.4. Design example is given to design the circuit elements of the amplifier.

Section 3.6 gives exact analysis of push-pull class-E amplifier with three modes of operation, using state equations. First order differential equations were written for the "three modes of operation" of the amplifier. These equations have been solved and steady-state circuit responses were plotted. Section 3.7 gives the harmonic analysis of output load current for three modes of operation. In section 3.8, analysis assuming constant input current and sinusoidal output current with three modes of operation is given. A comparison of the analysis of section 3.6 and the analysis of section 3.8 is also given. Section 3.9 deals with the design of class-E push pull amplifier for optimum operation at $D=\frac{2}{3}$. The design equations are derived from the optimum analysis of push-pull class-E amplifier for $D=\frac{2}{3}$. A design example is given to obtain circuit element values for optimum operation.

Chapter 2

Analysis & Design of Basic Class-E Tuned Amplifier

2.1 Introduction

2.1.1 Basic Criteria

The class-E power amplifier operation is a circuit topology, which consists of a switch and a load network, with three basic criteria imposed on voltage across the switch.

1. At the time of switch turn "on", the voltage across the switch should be zero.
2. The rate of change of voltage across the switch should also be zero, at the time switch turn "on".
3. At the time of switch turn "off", the voltage build-up across the switch should be delayed.

2.1.2 Basic Class-E Amplifier and Its Operation

The basic class-E amplifier circuit consists of a switch, a diode, a shunt capacitor, a series tuned circuit across a voltage source in series with a large inductor L_1 (RF Choke), as shown in Fig. 2.1. A switch driving signal has been assumed, which operates the switch, at desired operating frequency and duty-cycle. The capacitor C_1, includes any capacitance inherent in the switch S_1 and diode D_1, but its value is assumed independent of the voltage across it. Series inductor L_o and capacitor C_o may absorb any equivalent

inductance and capacitance of the load circuit respectively. The resistance R is sum of the, (1) input resistance of the load, (2) series a.c. resistances of L_o and C_o, (3) equivalent lumped resistance of the a.c. losses in shunt capacitor C_1, switch S_1 and diode D_1. The large inductor L_1 in series with voltage source provides approximately a constant D.C. current.

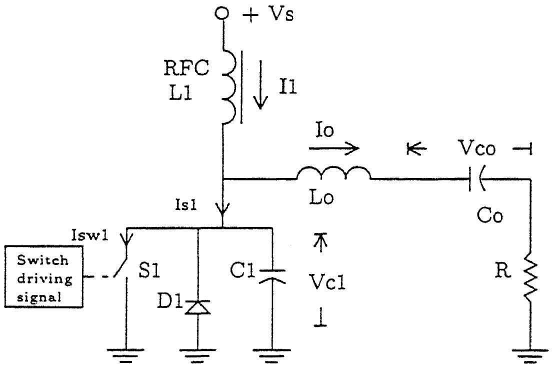

Figure 2.1: Basic class-E amplifier.

The switch driving signal operates the switch S_1 periodically. There are two modes of the circuit, one during switch "on", other during switch "off" time. At switch "on" mode; L_o, C_o and R forms a series resonant circuit and current source is shorted. During switch "off" mode; C_1, L_o, C_o and R forms a series resonant circuit and current source forces current through the network. The diode D_1 does not allow a negative voltage across the shunt capacitor C_1. A typical wave form for the two modes of the "optimum class-E operation" is shown in Fig. 2.2.

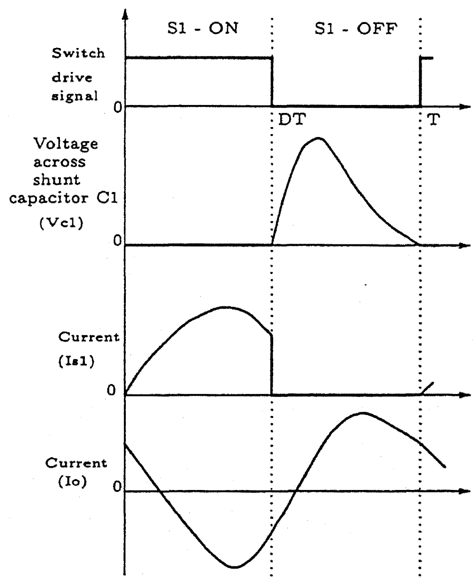

Figure 2.2: A typical waveform of optimum class-E amplifier.

2.1.3 Different Circuit Configurations of Class-E Amplifier

1. Class-E tuned power amplifier with shunt capacitor and series tuned output circuit is shown in Fig. 2.1. This circuit will be analyzed in section 2.2.

2. The push-pull configuration of the class-E amplifier is combination of two single-ended class-E amplifiers as shown in Fig. 2.3. This configuration provides symmetrical output waveforms and larger power output. The detail analysis of this configuration will be given in chapter-3.

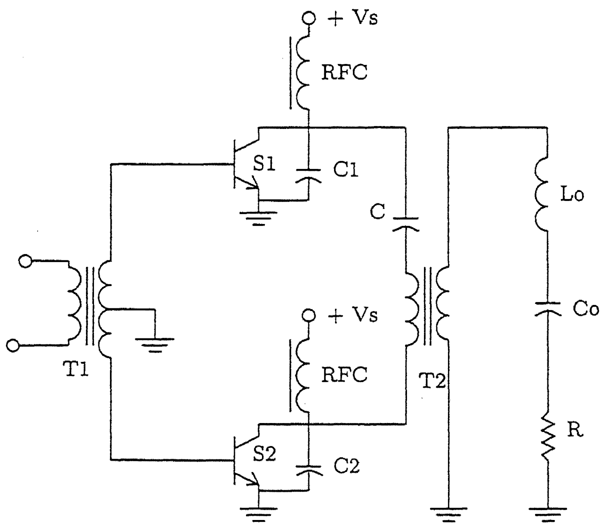

Figure 2.3: Push-pull class-E amplifier.

3. Class-E tuned power amplifier with only one inductor L and one capacitor C in load network is shown in Fig. 2.4. The resistor R is the load in which a.c. power is delivered. Principle of amplifeir operation and exact analysis is given in Ref. [4].

4. Class-E tuned amplifier configuration with finite D.C. feed inductance and no capacitance in parallel with switch is shown in Fig. 2.5. This configuration is suitable for low frequency applications (10 KHz to 500 KHz). L_1 is not an RF choke, but a finite inductance. The switch "off" operation is at zero magnitude and zero slope of switch current. But at the time of turn "on", finite voltage exists across the switch. Thyristor can also be used, instead of switching transistor. The detail analysis of this configuration has been given in Ref. [7].

5. Dual configuration of basic class-E amplifier is shown in Fig. 2.6. The series inductor L includes the effect of transformer. All circuit

elements and responses are dual of the circuit shown in Fig. 2.1. The switch must now be operated at zero current. In Fig. 2.1, the net effect of series-tuned circuit at operating frequency is inductive, in Fig. 2.6, the net effect of parallel tuned circuit at operating frequency will be capacitive. This circuit configuration is discussed in Ref. [2].

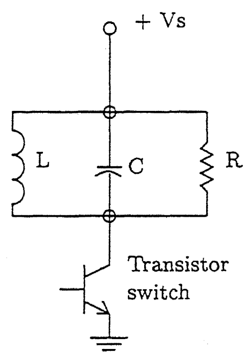

Figure 2.4: Class-E circuit with one inductor and one capacitor in load network.

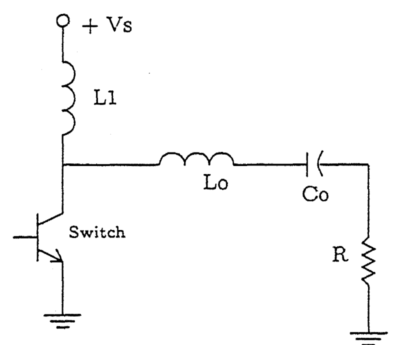

Figure 2.5: Class-E amplifier with finite DC feed inductance and no capacitance in parallel with switch.

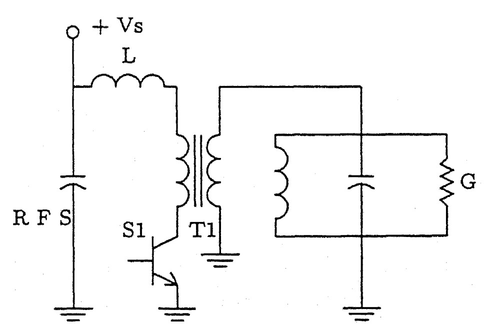

Figure 2.6: Dual of series tuned class-E amplifier with shunt capacitor.

6. Parallel tuned class-E amplifier with series capacitor is shown in Fig. 2.7. The analysis of this circuit is given in Ref. [2]. Waveforms of this circuit is given in Ref. [15].

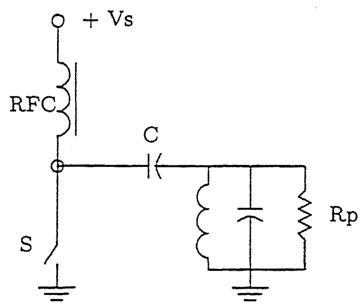

Figure 2.7: Parallel tuned class-E amplifier with series capacitor.

2.1.4 Application of Class-E Amplifier

Class-E circuit has been used in following applications.

1. High efficiency solid-state VHF-UHF tuned power amplifiers Ref. [3,13].
2. High frequency electric process heating Ref. [19,20,21,22].
3. Electronic lamp-ballast Ref. [27].
4. Zero voltage switching or zero current switching class-E rectifier Ref. [14,15,16,17,18].
5. Resonant regulated DC/DC power converters Ref.[23,24,25,26].

2.2 Analysis and Simulation of Basic Class-E Amplifier Using State Equations

2.2.1 Analysis

The basic class-E power amplifier to be analyzed is shown in Fig. 2.8. The analysis has been done with following assumptions,

1. The active device S_1 acts as an ideal switch i.e., zero "on" resistance, infinite "off" resistance, and zero switching time.
2. Diode has zero forward resistance, infinite reverse resistance and zero transition time from conduction state to blocking state or vice-versa.
3. Inductor L_1 (RF choke) is large enough to provide constant input current at steady-state operation.
4. All passive elements are ideal.

5. The shunt capacitance C_1 is independent of switch voltage and includes any capacitance inherent in switch and diode.

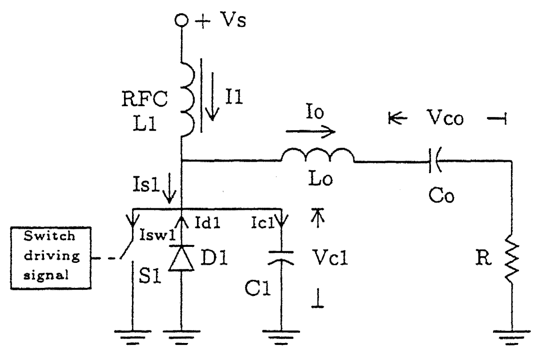

Figure 2.8: The class-E amplifier with diode.

The switch S_1 in Fig. 2.8 is operated periodically. The switch "on" interval is DT and "off" interval is (1-D)T. Where, D is switch duty-cycle and T is time-period of the switch operation. Hence, the circuit operates in following two modes.

Mode-1

Fig. 2.9 shows the equivalent circuit when switch S_1 is "on" or diode D_1 is "on", since the circuit includes the diode across the switch to allow reverse current. The current I_{s1}, which equals $(I_1 - I_o)$, is carried by switch S_1 if it is positive and by D_1 if it is negative. We have three first order simultaneous differential equations and voltage V_{c1} is zero for this mode.

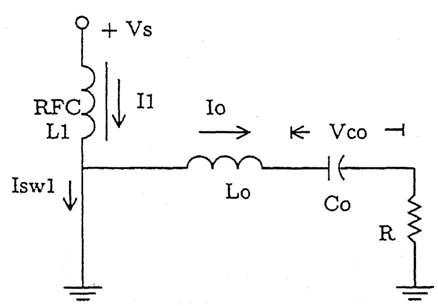

Figure 2.9: Equivalent circuit of the class-E amplifier for mode-1.

$$\frac{dI_1}{dt} = \frac{V_s}{L_1} \tag{2.1}$$

$$\frac{dI_o}{dt} = \frac{-V_{co}}{Lo} - \frac{RI_o}{L_o} \tag{2.2}$$

$$\frac{dV_{co}}{dt} = \frac{I_o}{C_o} \tag{2.3}$$

Mode-2

Fig. 2.10 shows, the equivalent circuit for this mode when switch is "off" and diode D_1 is reverse biased due to charge on C_1. For this state, we have four first order differential equations.

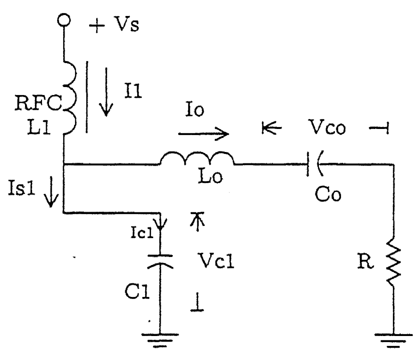

Figure 2.10: Equivalent circuit of the class-E amplifier for mode-2.

$$\frac{dI_1}{dt} = \frac{V_s - V_{c1}}{L_1} \tag{2.4}$$

$$\frac{dI_o}{dt} = \frac{V_{c1} - V_{co}}{L_o} - \frac{RI_o}{L_o} \tag{2.5}$$

$$\frac{dV_{co}}{dt} = \frac{I_o}{C_o} \tag{2.6}$$

$$\frac{dV_{c1}}{dt} = \frac{I_1 - I_o}{C_1} \tag{2.7}$$

Eq. (2.4), (2.5) and (2.6) are same as Eq. (2.1), (2.2) and (2.3) respectively with $V_{c1} = 0.0$. Therefore, only one set of equations need be programmed for simulation. If during switch "off" interval, the voltage $Vc1$ goes negative, the diode D_1 will conduct and circuit will revert to mode (1).

2.2.2 Circuit Parameters

Following circuit parameters have been chosen to simulate the circuit:

$$V_s = 100V,\ L_1 = 5mH,\ f = 50KHz,\ D = 0.5,\ R = 3.089\Omega,\ C_1 = 0.203\mu F,$$
$$C_o = 0.13\mu F,\ L_o = 90.13\mu H.$$

2.2.3 Method of Solution

Method of solution for obtaining steady-state characteristics by simulation is as following.

1. It is assumed that, the circuit starts with switch "on" at time t=0, the three first order simultaneous differential equations (2.1)—(2.3) are solved for interval between zero and DT with all initial conditions equal to zero.

2. Solutions of I_1, I_o, V_{co} and V_{c1} at time DT are taken as initial conditions to start solution for switch "off" interval from DT to T. For this interval four simultaneous differential equations (2.4)—(2.7) are solved.

3. Next cycle repeats with solutions of I_1, I_o, V_{co} and V_{c1} at the end of previous cycle, taking as initial conditions for the interval zero and DT. For interval DT and T step (2) is repeated.

The above solution steps (2) and (3) repeat till steady-state is achieved i.e., values of I_1, I_o, V_{co} and V_{c1} at time (n+1)DT and (n+1)T almost equals the respective values at nDT and nT. Where, n is the number of cycle of switch operations.

Flowchart and computer program used to simulate the class-E amplifier is given in appendix (A).

2.3 Simulation Study

2.3.1 Transient Solutions

Fig. 2.11 shows, how V_{c1} and I_o reaches from transient state to steady-state. With the circuit parameters given in section 2.2.2. It is shown that the steady-state is reached in around 400 cycles of operation. The plot shown is the curve for every tenth cycle. The circuit starts with switch-on for half time-period, then switch-off for next half time-period. At steady -state the peak value of V_{c1} is 358.75V and peaks of I_o are +34.92A and -31.38A. For optimum circuit values, maximum stress on the switch is the

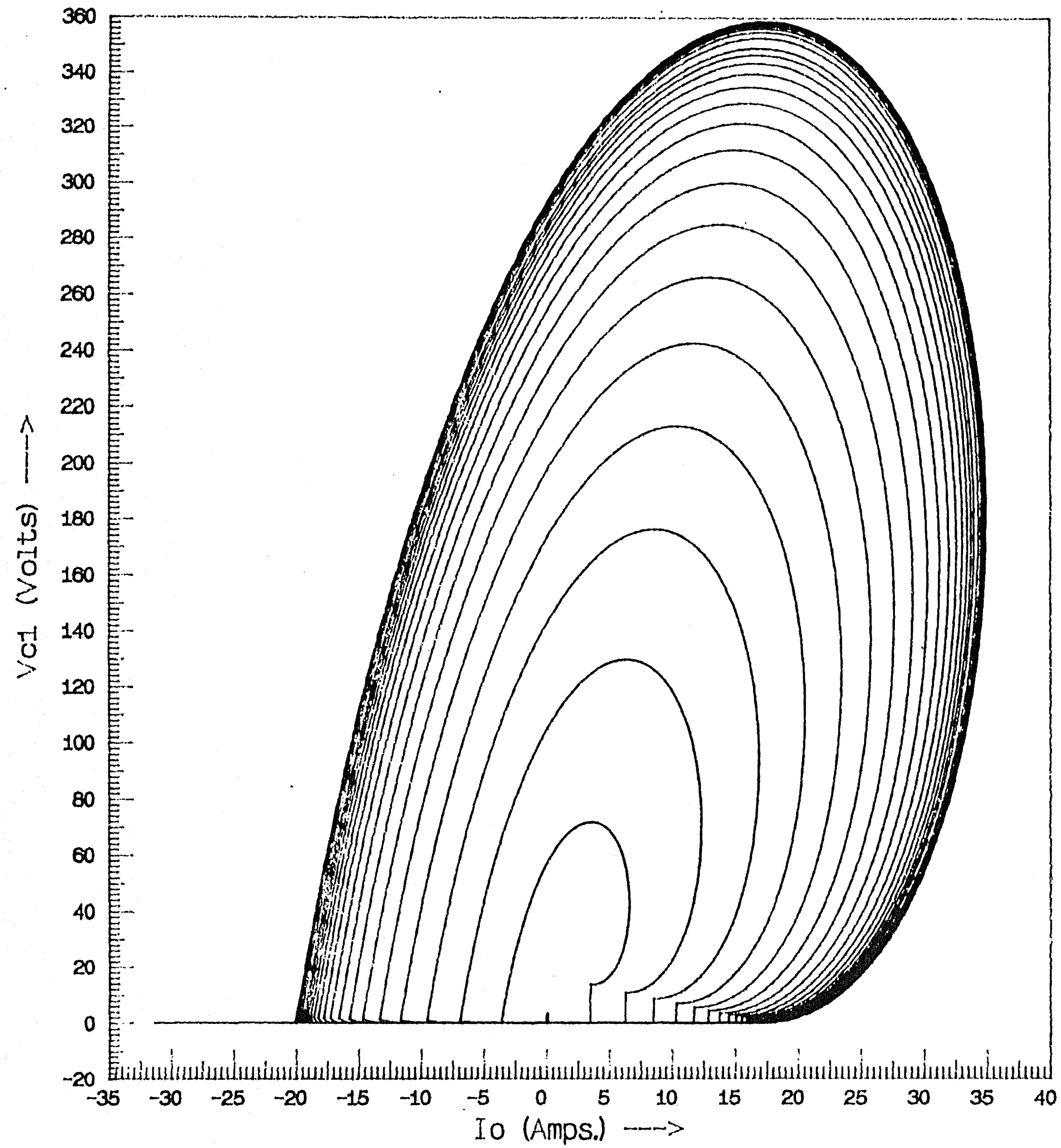

Figure 1.11: Transient build-up of Vc1 and Io.

peak value of V_{c1}. To start the circuit the supply voltage is applied at the same instant or after the switching signal starts.

Fig. 2.12 shows V_{c1} vs I_o curve at steady-state for one time-period. This figure is the outermost curve of Fig. 2.11. The switch turn-on occurs at t=0 sec., at this instant V_{c1} and its slope is zero. The output current equals to source current value. The I_o goes to its negative peak value and returns to a value close to source current in magnitude, but negative, at this instant switch is turned-off. The current I_o and voltage V_{c1} starts increasing. The capacitor voltage V_{c1} reaches to its peak value when current I_o equals the source current value I_1. Further, V_{c1} starts decreasing and I_o increases to a positive peak value. Now, both V_{c1} and I_o decreases, till the switch is turned-on. At switch turn-on, current I_o equals the source current I_1.

2.3.2 Steady-state Solutions

Taking the optimum values of the design components steady-state simulation results have been obtained. The curves of the steady-state solution shown in different plots have, the first half of the time-period as the switch "on" interval i.e., zero to DT and the second half of the time-period as switch "off" interval i.e., DT to (1-D)T.

Fig. 2.13 shows curve of voltage across the switch (V_{c1}) for one time-period at steady-state. It has been observed that at the time of switch "off" the shunt capacitor voltage V_{c1} starts rising parabolically up to 358.75V, and at the time of switch "on" the voltage V_{c1} decreases to zero, also the slope of V_{c1} becomes zero. Here, the advantage of zero voltage, zero slope turn "on" is that even if switching-on time is a significant fraction of switch time-period, the switching losses will be minimum.

Fig. 2.14 shows the curve of voltage across series capacitor C_o for one time period at steady-state. The maximum voltage is 905V and minimum voltage is -723V. The two peak values are unequal because the circuit configuration for the two halfs of time-period are different. The average voltage value of this waveform is equal to supply voltage V_s.

Fig. 2.15 shows plots of source current I_1 and the current which flows through either capacitor C_1 or switch S_1 for one time period. For optimum operation at steady-state the current through diode is zero. The source current I_1 is constant (17.35A approx) at steady-state, its magnitude depends on circuit parameters and magnitude of supply voltage source. At the time of switch turn "off" the switch current is slightly higher than twice of the source current (37.5A). This requires forced turn "off" of the switch. But at turn "on" the switch current starts rising from zero value. The peak value of the switch current $I_{s1_{pk}}$ is 48.8A.

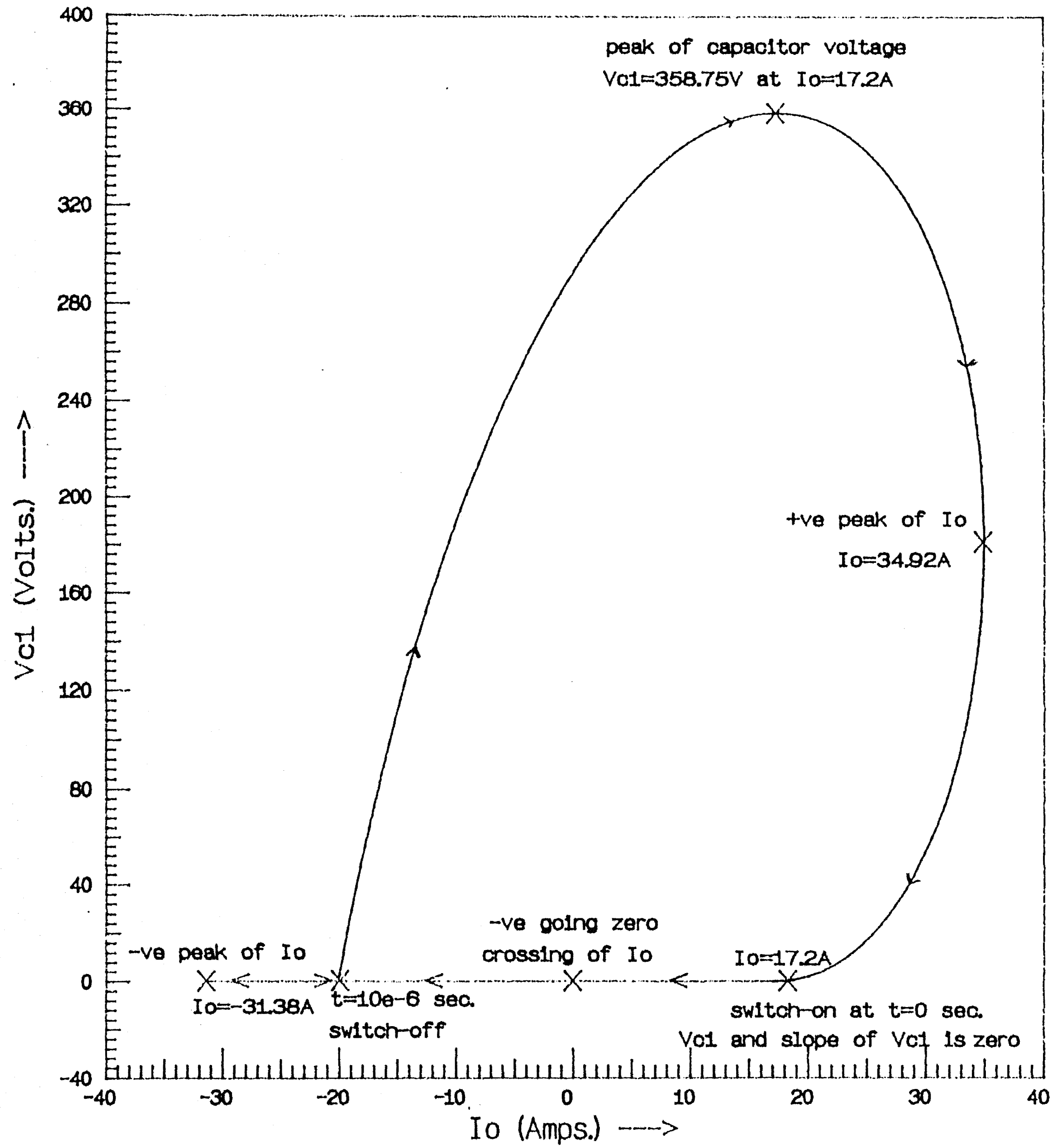

Figure 1.12: Steady-state curve of Vc1 vs Io.

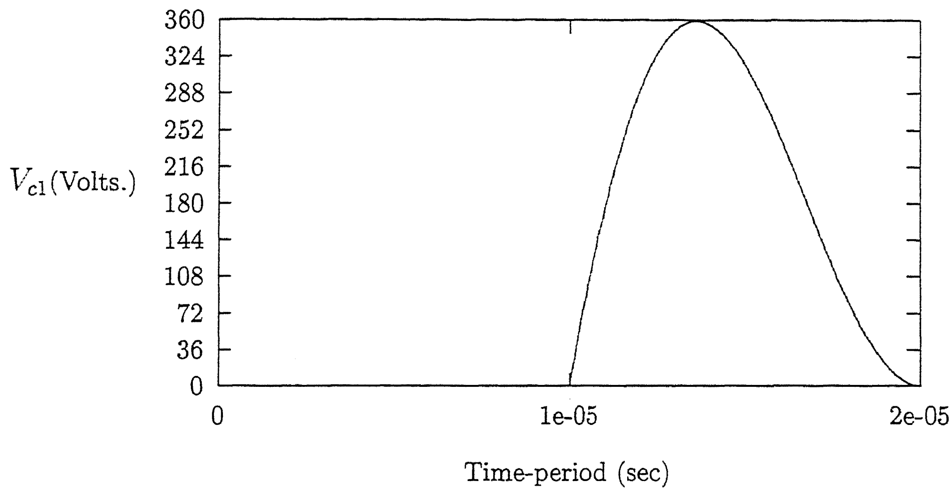

Figure 2.13: Switch voltage Vc1 vs time curve.

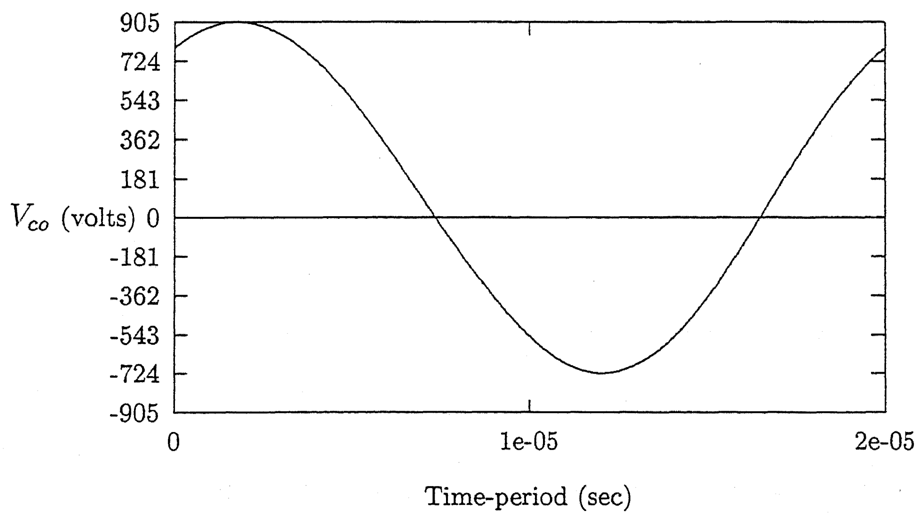

Figure 2.14: Series capacitor voltage vs time curve.

Fig. 2.16 shows output load current for one time- period. It is not pure sinusoidal. The two peaks of the current are not equal in magnitude. The two peak values are 34.92A and -31.38A.

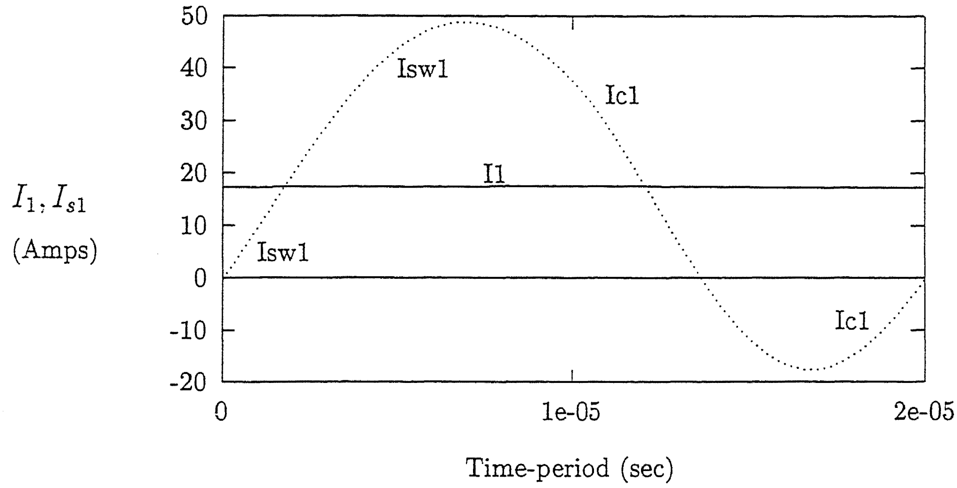

Figure 2.15: Constant source current (I1) and the current (Is1) which flows through the switch or capacitor.

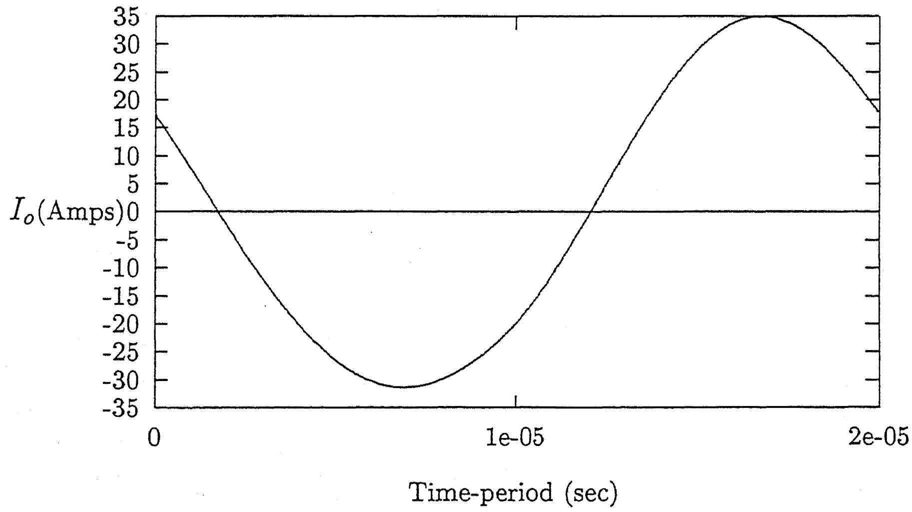

Figure 2.16: Output current for one cycle.

2.3.3 Study the Effect of Change in Power with Voltage Source

Fig. 2.17 shows the variation of power with change in supply voltage magnitude. Power follows the square law with source voltage V_s. Hence, the source voltage V_s and output current I_o has linear relation.

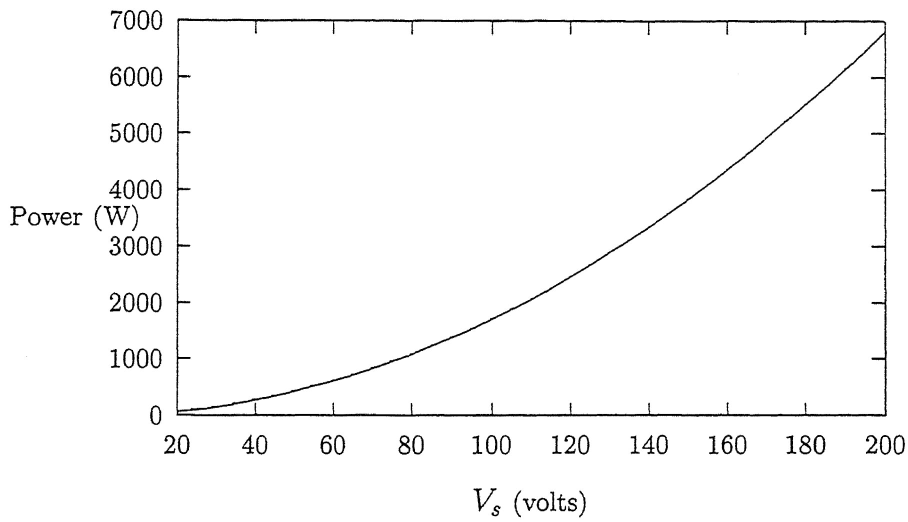

Figure 2.17: Supply voltage vs power output in load resistor.

2.3.4 Effect of Change in Load Resistance

Fig. 2.18 is plot of load resistance vs power output in the load. If the resistance R is decreased from its optimum value 3.089Ω to a value very near to zero, the power output in this will also decrease. If it is increased beyond the optimum value the power output decreases because shunt capacitor discharges at the time of turn "on".

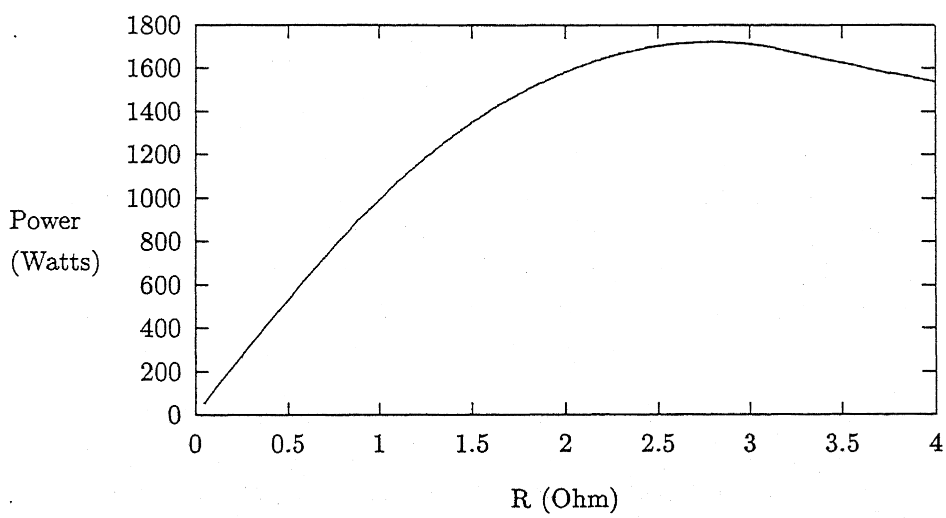

Figure 2.18: Curve for change in output power with output resistance.

Fig. 2.19 Shows shunt capacitor voltage V_{c1} for three resistance values. The plot is for one time-period at steady- state. If resistance R is decreased below the optimum value the circuit becomes underdamped, i.e., V_{c1} goes negative at the time of switch turn "on", but the diode D_1 starts conduction

and holds V_{c1} at zero (plot (a) with R=0.1Ω). The increase in resistance R beyond optimum value the circuit becomes overdamped, i.e., at the time of switch turn "on" V_{c1} is at some positive voltage. At the time of switch turn "on" V_{c1} discharges through switch, resulting in power loss.

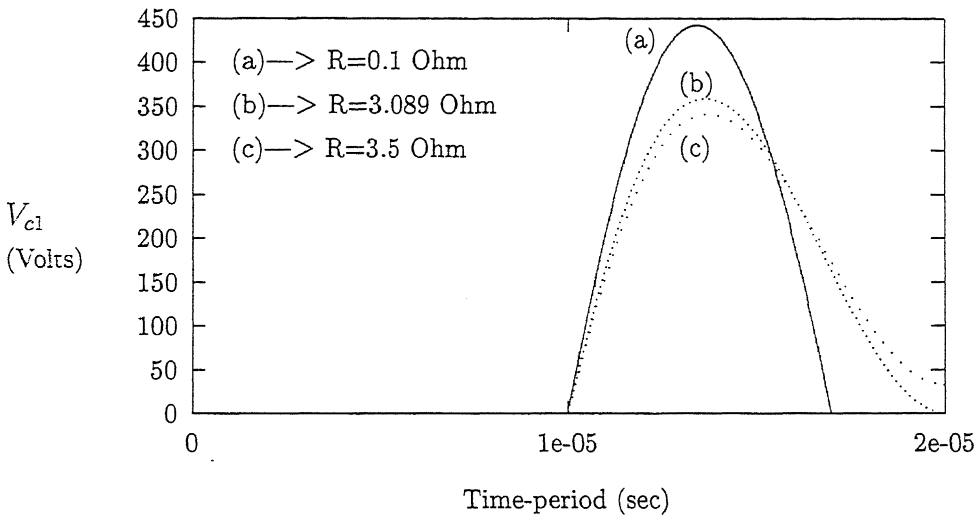

Figure 2.19: Variation of voltage across shunt capacitor with change in load resistance from its optimum value.

During optimum operation of class-E amplifier, the diode D_1 does not conduct at steady-state. Because at time of switch turn "on" the V_{c1} and its slope both are zero i.e., V_{c1} never goes to negative value. The efficiency of the circuit at optimum operation is nearly 100 percent.

During the suboptimum operation of the class-E amplifier, if V_{c1} is at some positive value at the time of switch turn "on", the diode never conducts during steady-state. But this forces the discharge of capacitor C_1 through switch S_1 at turn "on". This results in loss in energy and it may cause very large current through the switch, which can damage it. Since the circuit has ideal parameters, the loss which affects the efficiency is only the discharge of capacitor at the time of turn "on".

Suboptimum operation of class-E amplifier, in which voltage V_{c1} tends to go to negative, forces diode D_1 to conduct during steady-state. The switch is turned "on" at zero voltage, and switch current will start building up after the conduction of diode is over. The frequency, duty-cycle and load resistance variation is possible independently up to some extent without affecting the efficiency. The efficiency of the circuit is nearly 100 percent. Because the switch-on takes place at zero voltage and non-zero slope (finite turn-on current).

Fig. 2.20 is the plot of resistance vs peak voltage across shunt capacitor C_1. This plot helps in selecting the device voltage rating, where the load

resistance value changes (this is usual in induction heating loads). Here, the Maximum voltage value is peak of any voltage that occured from transient state to steady-state. The steady-state peak voltage value may be less than the peak voltage during transition from initial state to steady-state. For example, Fig. 2.19 shows peak V_{c1} = 442 V for R = 0.1 Ω, and Fig. 2.20 and 2.21 shows maximum V_{c1} 700 V.

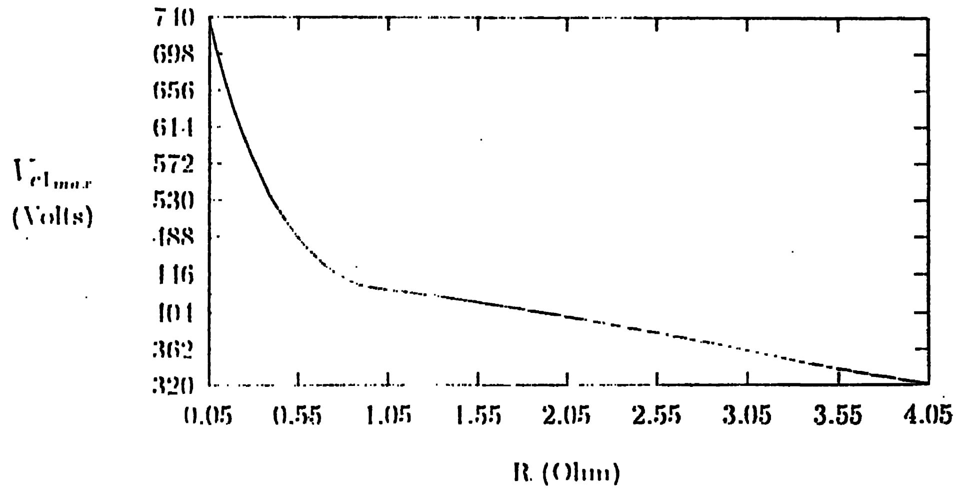

Figure 2.20: Peak voltage across switch during transients with different values of load resistance.

Fig. 2.21 shows the plot of peak shunt capacitor voltage $V_{c1_{max}}$ vs respective switching cycle. The plot is for first 1000 cycles. The circuit operation reaches steady-state in around 300 to700 cycles. It is observed that if the load resistance is decreased below a certain value (1.0 Ω for this circuit), the shunt capacitor voltage become underdamped. This puts very large voltage stress across the switch. This is taken care in selecting switch ratings.

2.3.5 Change in Load Resistance and Series Tuned Inductance

When the class-E amplifier is applied for power supply of high frequency induction heating, the change in load resistance R and change in load inductor L_o is very common. Here, a study has been done for this case. The load resistance is the resistance of induction heating "job". The inductance L_o includes the inductance of "job work". The resistance R generally increases with temperature, and effective inductance L_o generally decreases. Nature of change of R and L_o depends on the material and its way of processing

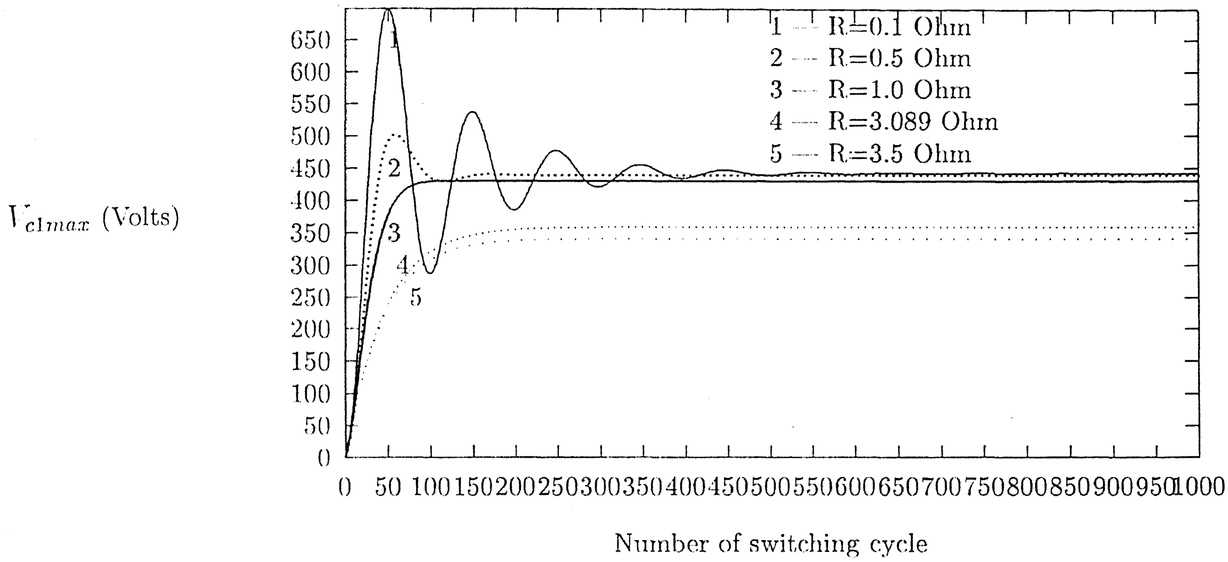

Figure 2.21: Peak voltage across switch during transients vs switch cycle for different load resistances.

with temperature. In the present study it is assumed that increase in R and decrease in L_o is linear, as shown in Fig. 2.22.

Fig. 2.22 shows the increase in resistance from 1Ω to 3Ω and decrease in L_o from $102\mu H$ to $90\mu H$. The range of change in R and L_o is chosen such that the switch turn "on" and turn "off" should be at zero voltage.

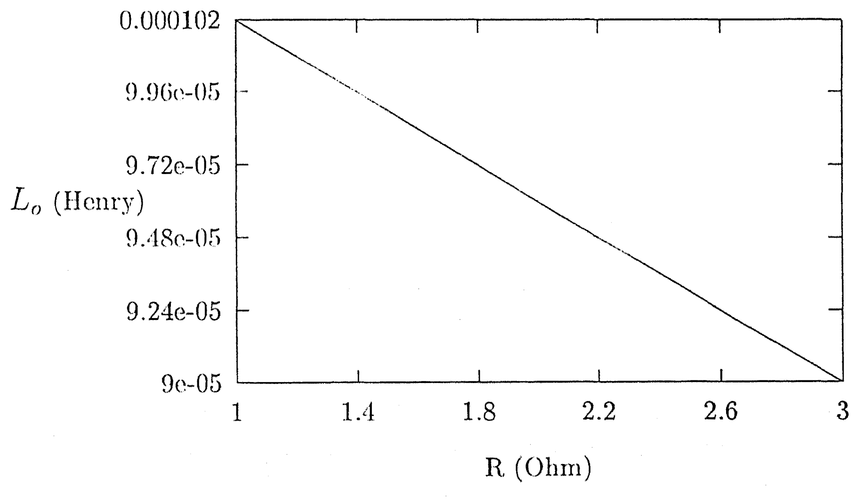

Figure 2.22: Assumed characteristics of Lo vs R with temperature during induction heating.

Change in power output is shown in Fig. 2.23 from operating point with $R = 1\ \Omega$, $L_o = 102\mu H$ to $R = 3\ \Omega$, $L_o = 90\mu H$. It is observed that the power output to the load will increase as resistance increases. The desired power output can be adjusted by changing supply voltage.

Fig. 2.24 shows evidence(e.g.,for three cases) that voltage across switch

will be zero both at turn "on" and at turn "off". This plot is for steady-state and one time-period.

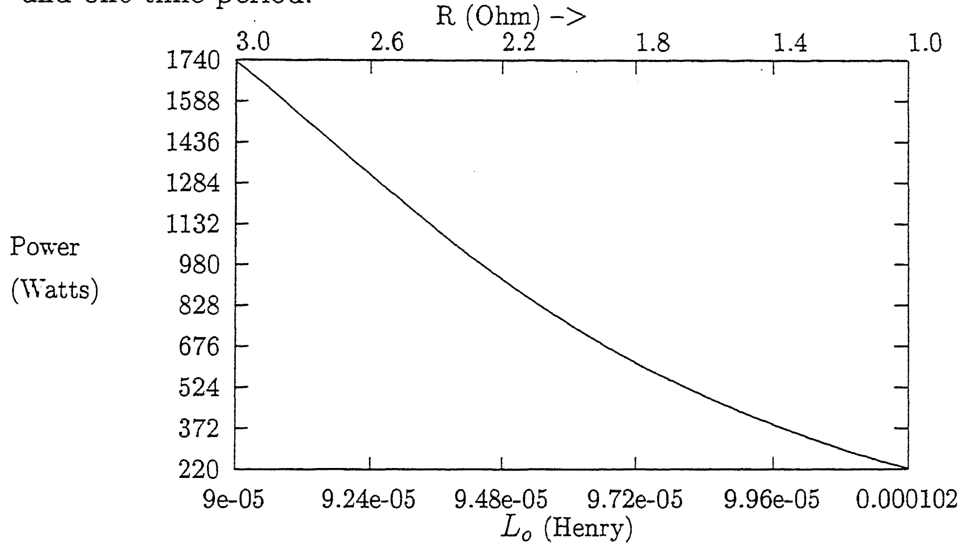

Figure 2.23: Change in output power due to change in Lo and R according to characteristic shown in Fig. 2.22.

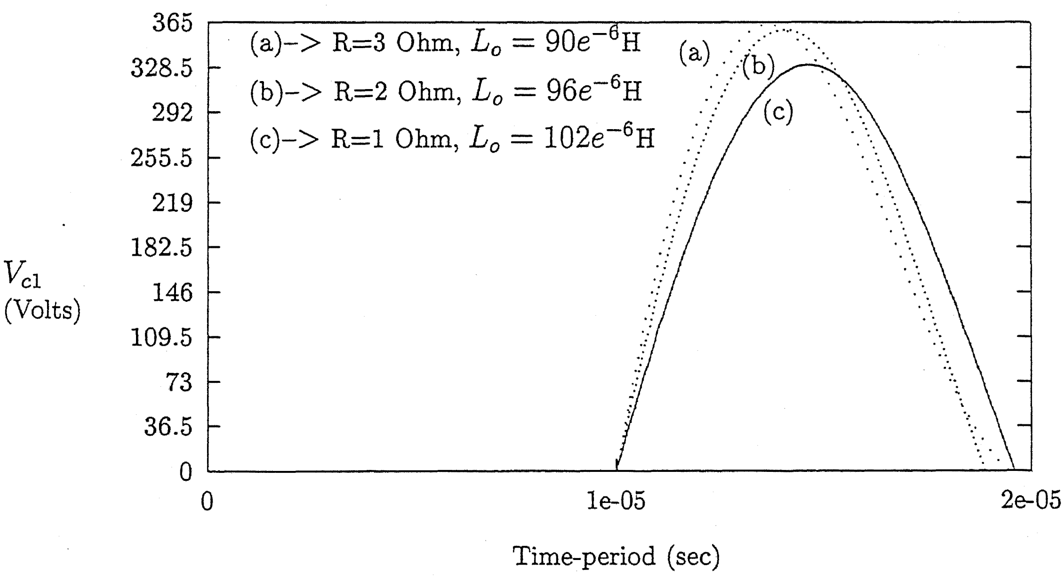

Figure 2.24: Voltage across the switch at steady-state with variation in R and Lo.

Fig. 2.25 provides help in selecting peak voltage rating of the device. This plot is for any peak voltage occured during transient state to steady-state transition. Although the steady-state peak voltage will be lower than the peak voltage during transient, the device should be capable of withstanding any peak voltage occuring across it at any time. Here, the variation of maximum voltage across the switch is plotted for transition from operating point with $R = 1\ \Omega$, $L_o = 102\mu H$ to $R = 3\ \Omega$, $L_o = 90\mu H$.

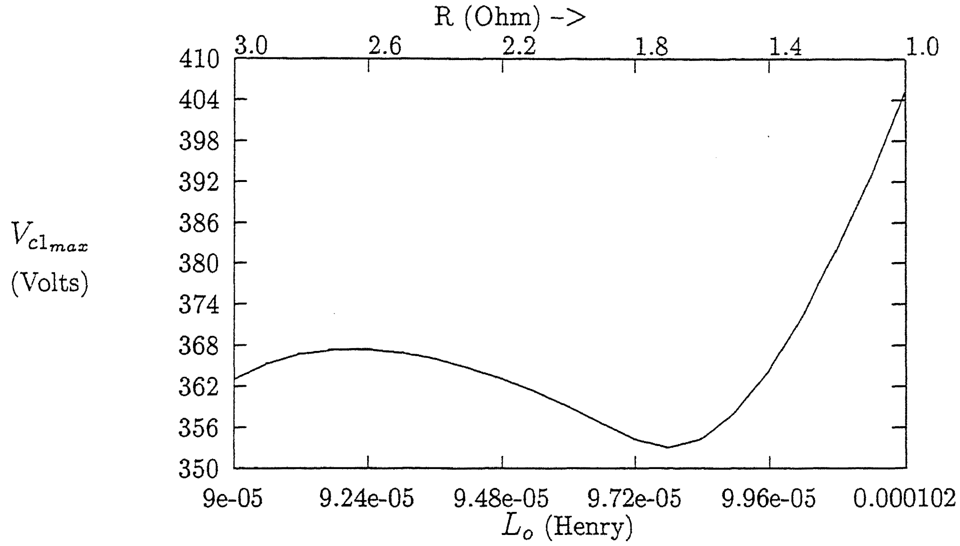

Figure 2.25: Peak transient voltage across the switch.

2.3.6 Change in Power with Change in Frequency

If all circuit element values are held constant, and resistance R is chosen 1.0 Ω, there is a range of frequency (48.5 KHz to 53.7 KHz) in which it can be varied without affecting efficiency of the tuned power amplifier. The small change in frequency results large change in power output as shown in Fig. 2.26. This can be applied in some induction heating loads, where change in resistance and change in inductance of work coil is negligible. This can also be used to regulate power output, if input supply voltage fluctuates.

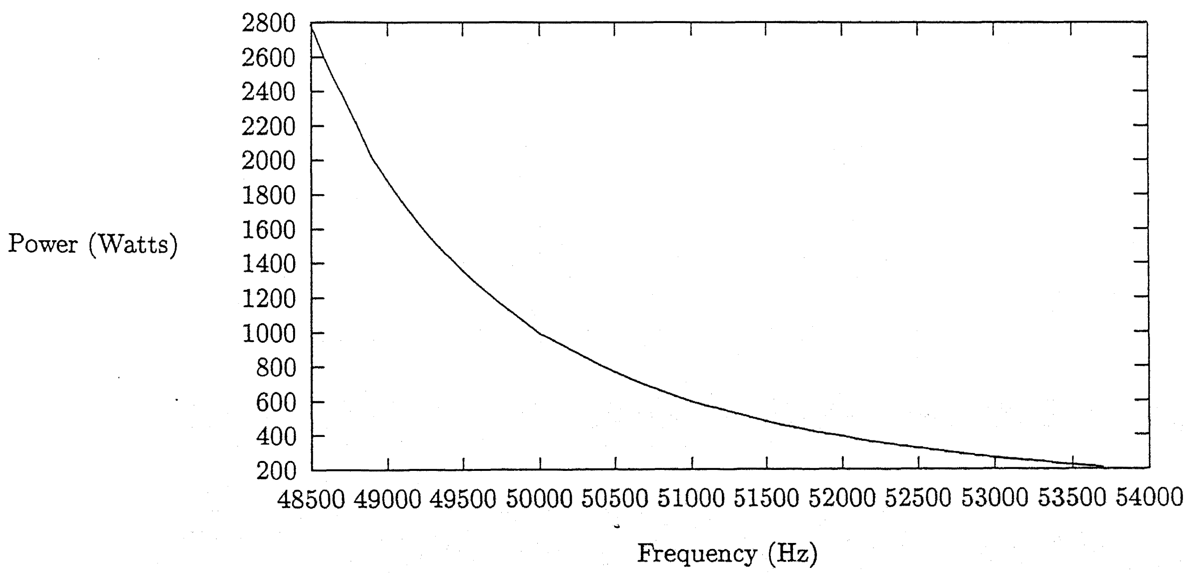

Figure 2.26: Change in power with frequency.

2.4 Harmonic Analysis

The class-E circuit is single ended and has two modes of operation. It may contain harmonics in its output waveform. To obtain harmonic components in output current I_o, and shunt capacitor voltage V_{c1}, we use discreate fourier transform method. To obtain harmonic components, we use NAG subroutine C06EAF. This subroutine is based on Discreate Fourier Transform.

2.4.1 Harmonic Analysis of Output Current

The harmonic components of output current I_o shown in Fig.2.16 for optimum case have been obtained. Peak magnitudes and phase angles of different harmonics are shown in table-2.1. It is observed that first three harmonics are significant, and rest of the harmonics are less than one percent of the fundamental. Phase angle of fundamental is 145.55 degree and peak magnitude is 33.15A. The phase angles of even harmonics are slowly decreasing from 221.19 degree, as the order of harmonics increases.

Table-2.1

Harmonics and phases of Output Current

Harmonics	Peak-magnitude (Amp.)	Phase angle (Degree)
1	33.146	145.55
2	1.925	221.19
3	0.294	-1.82
4	0.126	201.05
5	0.0562	-0.916
6	0.0335	194.33
7	0.0198	-0.443
8	0.0136	190.98
9	0.0092	-0.124
10	0.0068	189.01

2.4.2 Harmonic Analysis of Shunt Capacitor Voltage

The harmonic components of shunt capacitor voltage V_{c1} shown in Fig.2.13 for optimum case have been obtained. Magnitudes and phase angles of different harmonics are listed in table-2.2. The average of the shunt capacitor voltage is equal to the supply voltage. First three harmonics are significant, and rest are very small.

Table-2.2		
Harmonics and phases of Shunt Capacitor Voltage		
Harmonics	Peak-magnitude (Volt.)	Phase angle (Degree)
0	99.999	0.000
1	82.119	196.161
2	42.838	-52.980
3	11.301	85.659
4	6.748	-70.876
5	3.843	87.469
6	2.775	-77.125
7	1.934	88.215
8	1.521	-80.318
9	1.164	88.624
10	0.962	-82.252

The shunt capacitor voltage will always be equal to the voltage across the series tuned impedance Zo. Hence, the average D.C. voltage blocked by series capacitor Co will be equal to the supply voltage Vs. The different harmonic components of output current can also be obtained by dividing the different harmonic components of shunt capacitor voltage by series impedance Zoh. Where Zoh is for respective harmonic frequency.

2.5 Analysis Assuming Constant Input Current and Non-Sinusoidal Output Current

A simple equivalent circuit of class-E amplifier is shown in Fig. 2.27. Analysis of the circuit has been done with following assumptions.

1. The RF choke L_1 in series with supply voltage source provides approximately constant D.C. input current I_1.

2. The output current $I_o(\theta)$ is non-sinusoidal at the operating frequency of the amplifier. But average of the output current $I_o(\theta)$ is zero, at steady-state, because of output capacitor C_o. The quality-factor of the series tuned output circuit is finite.

3. The switch and diode have ideal characteristics.

4. All passive elements are ideal.

In the equivalent circuit shown in Fig. 2.27 the series reactance jX is produced by series connection of L_o and C_o (Fig. 2.1) at operating frequency of the tuned amplifier. If the operating frequency is equal to the resonant frequency of L_o and C_o the reactance jX equals zero.

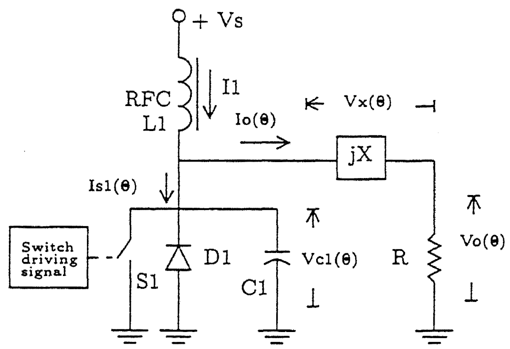

Figure 2.27: Equivalent circuit of class-E amplifier.

Let the voltage across resistance R be $V_o(\theta)$, voltage across jX is $V_x(\theta)$ and voltage across the shunt capacitor is $V_{c1}(\theta)$. The current through series tuned circuit is $I_o(\theta)$. Hence, current $I_{s1}(\theta)$ (shown in Fig. 1.27) is equal to the difference between the input current I_1 and output current $I_o(\theta)$.

We start the analysis, first by assuming the output current I_o pure sinusoidal, then we impose the conditions of optimum operating conditions to obtain the circuit parameters. Later a general analysis have been done by assuming harmonics in output current.

2.5.1 Analysis Assuming Output Current to be Fundamental Sine-Wave

If the quality factor of the series tuned circuit is very high the second and higher order harmonics of output current will be very small w.r.t. to fundamental component. The analysis assuming only fundamental component in output current is simple but approximate.

Suppose, the sinusoidal current $I_o(\theta)$ is given by,

$$I_o(\theta) = I_{om} \sin(\theta + \phi) \tag{2.8}$$

Where, I_{om} is the peak-current value, ϕ is the phase angle w.r.t. to some arbitrary reference, and $\theta(=\omega t)$ is angle in radian corresponding to operating frequency of tuned amplifier. The instant when switch is turned "on" is taken as the reference for phase of current $I_o(\theta)$ (Fig. 2.2).

When switch S_1 is "off", capacitor C_1, equivalent reactance jX and resistance R are in series. $V_{c1}(\theta)$ is given by,

$$V_{c1}(\theta) = V_x(\theta) + V_o(\theta) \tag{2.9}$$

Hence, from Eq. (2.8) and (2.9), we have

$$V_{c1}(\theta) = I_{om}\sqrt{X^2 + R^2}\sin(\theta + \phi + \psi) \tag{2.10}$$

Where, $\psi = \arctan(X/R)$.

The voltage $V_{c1}(\theta)$ exists only during switch is "off", and diode D_1 is not conducting. The average voltage across the source inductor L_1 is zero, because it carries a constant D.C. current. Hence, the average voltage across the shunt capacitor C_1, will be equal to supply voltage.

The circuit model of the class-E tuned amplifier, chosen for approximate analysis contains all lossless circuit elements. The loss occurs only in condition if the switch turn "on" takes place at some finite capacitor voltage value. The class-E circuit will operate at 100 percent efficiency, if the switch voltage drops to zero at the instant of the switch turn "on".

Power output increases as duty-cycle is increased, but the "peak-power output capability" for a given switching device can be achieved by operating the class-E amplifier at 50-percent duty-cycle and at optimum operating conditions, Ref.[2]. At optimum operating point voltage across the switch does not become negative, hence, the circuit will work without diode D_1. For very high frequency operation, if speed of diode is slower than the the switch speed, the optimum operating point is chosen.

For the analysis of class-E tuned amplifier at optimum operation, we will choose duty-cycle D=0.5 and zero magnitude and zero slope of switch voltage at switch turn "on".

The switch S_1 is "on" for time duration zero to DT (i.e., 0 to π) and it is "off" from DT to T (i.e., π to 2π). $V_{c1}(\theta)$ is non zero during switch "off" time (Fig. 2.2), and current I_{s1} will flow only through the capacitor, because diode D_1 will not conduct at optimum operation. Equivalent circuit of Fig. 2.10 applies to the switch "off" interval.

Hence, $I_{s1}(\theta)$ is given by,

$$I_{s1}(\theta) = I_1 - I_{om}\sin(\theta + \phi) \tag{2.11}$$

Also, voltage $V_{c1}(\theta)$ is given by,

$$V_{c1}(\theta) = \frac{1}{wC_1}\int_{\pi}^{\theta}[I_1 - I_{om}\sin(\alpha+\phi)]d\alpha \tag{2.12}$$

Where angle θ can be any value between π to 2π. The Eq. (2.12) can be written as,

$$V_{c1}(\theta) = \frac{1}{wC_1}[I_1(\theta-\pi) + I_{om}\cos(\theta+\phi) + I_{om}\cos\phi] \tag{2.13}$$

The voltage across inductor equal V_s, when switch S_1 is closed and $(V_s - V_{c1})$ when the switch is open. In order that the average inductor voltage be zero in steady-state, the average value of V_{c1} must equal V_s. Therefore

$$V_s = \frac{1}{2\pi}\int_{\pi}^{2\pi} V_{c1}(\theta)d\theta \tag{2.14}$$

Using Eq. (2.13) and (2.14), we get the following result.

$$V_{c1av} = \frac{1}{4\pi wC_1}[I_1\pi^2 + 4I_{om}\sin\phi + 2\pi I_{om}\cos\phi] \tag{2.15}$$

Average value of $V_{c1}(\theta)$ is equal to V_s,

$$V_s = \frac{1}{4\pi wC_1}[I_1\pi^2 + 4I_{om}\sin\phi + 2\pi I_{om}\cos\phi] \tag{2.16}$$

Using Eq. (2.13), we obtain fundamental frequency component of $V_{c1}(\theta)$, as below,

$$\begin{aligned} V_{c1fundamental}(\theta) &= \frac{1}{\pi wC_1}[\{-I_1\pi - 2I_{om}\cos\phi - \frac{\pi I_{om}}{2}\sin\phi\}\sin\theta \\ &\quad +\{2I_1 + \frac{\pi I_{om}}{2}\cos\phi\}\cos\theta] \end{aligned} \tag{2.17}$$

From Eq. (2.10), we have,

$$V_{c1}(\theta) = I_{om}Z_o\cos(\phi+\psi)\sin\theta + I_{om}Z_o\sin(\phi+\psi)\cos\theta \tag{2.18}$$

The fundamental frequency component of $V_{c1}(\theta)$ of Eq. (2.17) is equal to the voltage drop across $Z_o(=\sqrt{X^2+R^2})$. Hence, from Eq. (2.17) and

(2.18), we equate coefficients of $\sin\theta$ and $\cos\theta$, respectively and obtain following two equations;

$$(wC_1X - \frac{1}{2})I_{om}\sin\phi - (\frac{2}{\pi} + wC_1R)I_{om}\cos\phi = I_1 \tag{2.19}$$

$$wC_1RI_{om}\sin\phi + (-\frac{1}{2} + wC_1X)I_{om}\cos\phi = \frac{2I_1}{\pi} \tag{2.20}$$

From Eq. (2.19) and (2.20), we have

$$I_{om}\sin\phi = \frac{I_1[\frac{2wC_1R}{\pi} + wC_1X + \frac{4}{\pi^2} - \frac{1}{2}]}{(wC_1X - \frac{1}{2})^2 + wC_1R(\frac{2}{\pi} + wC_1R)} \tag{2.21}$$

$$I_{om}\cos\phi = \frac{I_1[\frac{2wC_1X}{\pi} - wC_1R - \frac{1}{\pi}]}{(wC_1X - \frac{1}{2})^2 + wC_1R(\frac{2}{\pi} + wC_1R)} \tag{2.22}$$

From Eq. (2.21) and (2.22),we have,

$$\phi = \arctan\frac{[R + \frac{\pi X}{2} + \frac{2}{\pi wC_1} - \frac{\pi}{4wC_1}]}{[X - \frac{\pi R}{2} - \frac{1}{2wC_1}]} \tag{2.23}$$

Generally, value of ϕ obtained by Eq. (2.23) will be in either in first quadrant or in fourth quadrant. If $I_{om}\cos\phi$ is negative, π radian is added to ϕ to make it in second or third quadrant.

If current source I_1 is realized by a very large inductor in series with voltage source Vs, the value of I_1 can be obtained from Eq. (2.16), because $I_{om}\sin\phi$ and $I_{om}\cos\phi$ are known.

Once, I_1, Iom, and ϕ are known, the output current $I_o(\theta)$ and voltage $V_{c1}(\theta)$ can be found at any instant of time.

From Eq. (2.13) at $\theta = 2\pi$, we get, the value of $V_{c1}(\theta)$ as,

$$V_{c1}/_{\theta=2\pi} = \frac{1}{wC_1}(I_1\pi + 2I_{om}\cos\phi) \tag{2.24}$$

Consider, peak value of $V_{c1}(\theta)$ occurs at $\theta = \theta_{vmax}$, from Eq.(2.13), we get,

$$\frac{dV_{c1}(\theta)}{d\theta}/_{\theta=\theta_{vmax}} = 0 = I_1 - I_{om}\sin(\theta_{vmax} + \phi) \tag{2.25}$$

Since, values of I_1, I_{om} and ϕ are known, we can solve the value of θ_{vmax} from Eq. (2.25). Hence, V_{c1max} can be obtained from Eq. (2.13), using $\theta = \theta_{vmax}$.

Solution for Optimum Case

At the time of switch turn "on", the $V_{c1}(\theta)$ should be zero. Hence, from Eq. (2.13) at turn "on",

$$V_{c1}(\theta)/_{\theta=2\pi} = 0 = [I_1(2\pi - \pi) + I_{om}\cos(2\pi + \phi) + I_{om}\cos\phi] \qquad (2.26)$$

which gives,

$$\cos\phi = -\frac{\pi I_1}{2I_{om}} \qquad (2.27)$$

The slope of $V_{c1}(\theta)$ at $\theta = 2\pi$ must also be zero. Hence, again from Eq. (2.13),

$$\frac{dV_{c1}(\theta)}{d\theta}/_{\theta=2\pi} = 0 = [I_1 - I_{om}\sin(2\pi + \phi)] \qquad (2.28)$$

Eq. (2.28) gives,

$$\sin\phi = \frac{I_1}{I_{om}} \qquad (2.29)$$

Eq. (2.27) and (2.29) gives,

$$\phi = 2.57468^c \qquad (2.30)$$

From equation (2.16), (2.27) and (2.29), we obtain,

$$I_1 = \pi w C_1 V_s \qquad (2.31)$$

The output power in the load resistor is given by,

$$P_o = \frac{I_{om}^2 R}{2} \qquad (2.32)$$

The input power is simply,

$$P_i = V_s I_1 \tag{2.33}$$

Hence, efficiency is given by,

$$\eta = \frac{I_{om}^2 R}{2V_s I_1} \tag{2.34}$$

The maximum possible switch current is given by,

$$I_{s1(pk)} = I_1 + I_{om} \tag{2.35}$$

From equations (2.29), (2.30) and (2.35), we get,

$$I_{s1(pk)} = 2.862 I_1 \tag{2.36}$$

From equation (2.13) , at peak value of $V_{c1}(\theta)$,

$$\frac{dV_{c1}(\theta)}{d\theta}/_{\theta=\theta_{vmax}} = 0 = I_1 - I_{om}\sin\left(\theta_{vmax} + \phi\right) \tag{2.37}$$

From equations (2.29),(2.30) and (2.37), we obtain,

$$\theta_{vmax} = 4.27539^c \tag{2.38}$$

Substituting value of θ_{vmax} for θ in Eq. (2.13) and using Eq. (2.29), (2.30) and (2.31), we have $V_{c1}(\theta)$ at θ_{max} as,

$$V_{c1max} = 3.562 V_s \tag{2.39}$$

2.5.2 Analysis Assuming Output Current to be Non-Sinusoidal

Suppose, the output current $I_o(\theta)$ is non-sinusoidal and is represented by sum of its k–harmonic components (where, k is an integer and $k \to \infty$),as

$$I_o(\theta) = \sum_{n=1}^{k} Iom_n \sin(n\theta + \phi_n) \tag{2.40}$$

Where, Iom_n is the peak current value, and ϕ_n is the phase angle of n^{th} harmonic current, w.r.t. to some arbitrary reference, and $\theta(=wt)$ is angle in radian. The instant when switch is turned-on is taken as the reference for phase of current $I_o(\theta)$ (ref. Fig. 2.2). From Fig. 2.27, $V_{c1}(\theta)$ is given by,

$$V_{c1}(\theta) = V_x(\theta) + V_o(\theta) \tag{2.41}$$

The equivalent reactance for n^{th} harmonic will be jX_n. Hence, from Eq. (2.40) and (2.41), we have,

$$V_{c1}(\theta) = V_{c1_{av}} + \sum_{n=1}^{k} Iom_n\sqrt{X_n^2 + R^2}\sin(n\theta + \phi_n + \psi_n) \tag{2.42}$$

where, $\psi_n = \arctan\left(\frac{X_n}{R}\right)$.

The current $I_{s1}(\theta)$ (ref. Fig. 2.27) is given by,

$$I_{s1}(\theta) = I_1 - \sum_{n=1}^{k} Iom_n \sin(n\theta + \phi_n) \tag{2.43}$$

Also, voltage $V_{c1}(\theta)$ is given by,

$$V_{c1}(\theta) = \frac{1}{wC_1}\int_{\pi}^{\theta}[I_1 - \sum_{n=1}^{k} Iom_n \sin(n\alpha + \phi_n)]d\alpha \tag{2.44}$$

This voltage $V_{c1}(\theta)$ exists only when switch is off. From Eq. (2.44), we have,

$$V_{c1}(\theta) = \frac{1}{wC_1}[I_1(\theta - \pi) + \sum_{n=1}^{k}\frac{Iom_n}{n}\{\cos(n\theta + \phi_n) - (-1)^n\cos\phi_n\}] \tag{2.45}$$

The average inductor voltage is zero at steady-state, hence, the average value of V_{c1} will be equal to Vs. Therefore,

$$V_s = \frac{1}{2\pi}\int_{\pi}^{2\pi} V_{c1}(\theta)d\theta \tag{2.46}$$

Using Eq. (2.45) and (2.46), we have,

$$V_s = \frac{1}{2\pi wC_1}[\frac{\pi^2}{2}I_1 + \sum_{n=1}^{k} Iom_n\{\frac{1-(-1)^n}{n^2}\sin\phi_n - \frac{(-1)^n\pi}{n}\cos\phi_n\}] \quad (2.47)$$

If the impedance of series tuned output circuit is Zo_m, for m—harmonics, we can obtain $V_{c1}(\theta)$, using Eq. (2.40),

$$V_{c1}(\theta) = Vco_{av} + \sum_{m=1}^{k} Iom_m Zo_m \sin(m\theta + \phi_m + \psi_m) \quad (2.48)$$

For m^{th} harmonic component, $\psi_m = \arctan(\frac{X_m}{R})$; and average voltage across series capacitor Co is $V_{co_{av}}$ (where, $Vco_{av} = V_s$). From Eq. (2.45), $V_{c1}(\theta)$ can be represented by sum of its Fourier Components, as

$$V_{c1}(\theta) = Vc1_{av} + \sum_{m=1}^{k}[a_m\cos(m\theta) + b_m\sin(m\theta)] \quad (2.49)$$

Where, $V_{c1_{av}}$ is equal to supply voltage Vs, and

$$\begin{aligned} a_m &= \frac{1}{\pi wC_1}[\frac{\{1-(-1)^m\}}{m^2}I_1 + \frac{\pi}{2m}Iom_m\cos\phi_m \\ &+ \sum_{n=1,n\neq m}^{k}\frac{\{(-1)^{m+n}-1\}}{m^2-n^2}Iom_n\sin\phi_n] \end{aligned} \quad (2.50)$$

$$\begin{aligned} b_m &= \frac{1}{\pi wC_1}[-\frac{\pi}{m}I_1 - \frac{\pi}{2m}Iom_m\sin\phi_m + \sum_{n=1,n\neq m}^{k}\frac{m\{(-1)^{m+n}-1\}}{n(m^2-n^2)}Iom_n\cos\phi_n \\ &+ \sum_{n=1}^{k}\frac{(-1)^n\{1-(-1)^m\}}{mn}Iom_n\cos\phi_n] \end{aligned} \quad (2.51)$$

Eq. (2.48) can also be written as,

$$\begin{aligned} V_{c1}(\theta) &= V_{c1_{av}} + \sum_{m=1}^{k}[\{Zo_m Iom_m\sin(\phi_m+\psi_m)\}\cos(m\theta) \\ &+ \{Zo_m Iom_m\cos(m\phi+\psi_m)\}\sin(m\theta)] \end{aligned} \quad (2.52)$$

Where, $V_{c1_{av}}$ is equal to Vs. Since, Eq. (2.49) and (2.52) represent the same waveform, hence,

$$a_m = Zo_m Iom_m \sin(\phi_m + \psi_m) \tag{2.53}$$

$$b_m = Zo_m Iom_m \cos(\phi_m + \psi_m) \tag{2.54}$$

In Eq. (2.40), output current $I_o(\theta)$, consists of k—harmonic components. Hence, there will be 2k unknowns i.e., k—peak currents Iom_n, and k—phase angles ϕ_n. If we choose, k—harmonic components of $V_{c1}(\theta)$ (i.e., m=1 to k), from Eq. (2.50), (2.51), (2.53) and (2.54), we will have 2k equations. These equations can be written in the form of linear equations, with assuming $(Iom_n \sin\phi_n)$ and $(Iom_n \cos\phi_n)$ as unknowns.

Using Eq. (2.50) and (2.53), the m^{th} equation can be written as,

$$(\pi w C_1 R) Iom_m \sin\phi_m - \sum_{n=1,n\neq m}^{k} \frac{(-1)^{m+n} - 1}{m^2 - n^2} Iom_n \sin\phi_n$$
$$+(\pi w C_1 X_m - \frac{\pi}{2m}) Iom_m \cos\phi_m = \frac{1-(-1)^m}{m^2} I_1 \tag{2.55}$$

Using Eq. (2.51) and (2.54), the $(m+k)^{th}$ equation can be written as,

$$(\frac{\pi}{2m} - \pi w C_1 X_m) Iom_m \sin\phi_m + [\pi w C_1 R - \frac{(-1)^m\{1-(-1)^m\}}{m^2}] Iom_m \cos\phi_m$$
$$- \sum_{n=1,n\neq m}^{k} [\frac{m[(-1)^{m+n} - 1]}{n(m^2 - n^2)} + (-1)^n \frac{1-(-1)^m}{mn}] Iom_n \cos\phi_n = -\frac{\pi}{m} I_1 \tag{2.56}$$

Eq. (2.55) and (2.56) each forms (m=1 to k) k–equations. Hence, there are 2k equations and 2k unknowns (i.e., $Iom_n \sin\phi_n$ and $Iom_n \cos\phi_n$ for n=1 to k). Hence, all peak current values Iom_n and ϕ_n can be solved for given value of operating frequency, input current source value I_1 and circuit elements.

If current I_1 is unknown, we will have total (2k+1) unknowns. Thus, we will have to include one more equation i.e., Eq. (2.47). Hence, from Eq. (2.55), (2.56) and (2.47); we can write following matrix equation:

$$\begin{bmatrix} X_m & A_{mn} & B_{mn} \\ Y_m & C_{mn} & D_{mn} \\ H & U_n & V_n \end{bmatrix} \begin{bmatrix} I_1 \\ Iom_m \cos\phi_m \\ Iom_m \sin\phi_m \end{bmatrix} = \begin{bmatrix} E_m \\ F_m \\ G \end{bmatrix} \tag{2.57}$$

Where,

$$X_m = \begin{bmatrix} x_1 \\ \vdots \\ x_k \end{bmatrix}; \quad Y_m = \begin{bmatrix} y_1 \\ \vdots \\ y_k \end{bmatrix}; \quad U_n = \begin{bmatrix} u_1 \dots u_k \end{bmatrix};$$

$$V_n = \begin{bmatrix} v_1 & \dots & v_k \end{bmatrix}; \quad E_m = \begin{bmatrix} e_1 \\ \vdots \\ e_k \end{bmatrix}; \quad F_m = \begin{bmatrix} f_1 \\ \vdots \\ f_k \end{bmatrix}$$

$$A_{mn} = \begin{bmatrix} a_{11} & \dots & a_{1n} & \dots & a_{1k} \\ \vdots & & \vdots & & \vdots \\ a_{m1} & \dots & a_{mn} & \dots & a_{mk} \\ \vdots & & \vdots & & \vdots \\ a_{k1} & \dots & a_{kn} & \dots & a_{kk} \end{bmatrix}$$

$$B_{mn} = \begin{bmatrix} b_{11} & \dots & b_{1n} & \dots & b_{1k} \\ \vdots & & \vdots & & \vdots \\ b_{m1} & \dots & b_{mn} & \dots & b_{mk} \\ \vdots & & \vdots & & \vdots \\ b_{k1} & \dots & b_{kn} & \dots & b_{kk} \end{bmatrix}$$

C_{mn} and D_{mn} are also $k \times k$ matrices.

Here, both m and n vary from 1 to k. where, k is number of harmonics considered. Different elemental values of matrix is given as below.

For m=1 to k,

$$\begin{aligned} \mathcal{X}_m = -\tfrac{\{1-(-1)^m\}}{m^2}; &\quad \mathcal{E}_m = 0.0 \\ H = \tfrac{\pi^2}{2}; &\quad \mathcal{Y}_m = \tfrac{\pi}{m} \\ G = 2\pi w C_1 V_s; &\quad \mathcal{F}_m = 0.0 \end{aligned}$$

For n=1 to k,

$$\mathcal{U}_n = \frac{1-(-1)^n}{n^2}$$

$$
\begin{aligned}
V_n &= -\frac{(-1)^n \pi}{n} \\
a_{mn}\ \cancel{A_{mn}} &= -\frac{\{(-1)^{m+n}-1\}}{m^2-n^2}, for\ n \neq m. \\
&= \pi w C_1 R,\ for\ n = m. \\
b_{mn}\ \cancel{B_{mn}} &= 0.0,\ for\ n \neq m. \\
&= \pi w C_1 X_m - \frac{\pi}{2m},\ for\ n = m. \\
c_{mn}\ \cancel{C_{mn}} &= 0.0,\ for\ n \neq m. \\
&= \frac{\pi}{2m} - \pi w C_1 X_m,\ for\ n = m. \\
d_{mn}\ \cancel{D_{mn}} &= -[\frac{m\{(-1)^{m+n}-1}{n(m^2-n^2)} + (-1)^n \frac{1-(-1)^m}{mn}],\ for\ n \neq m. \\
&= [\pi w C_1 R - (-1)^m \frac{1-(-1)^m}{m^2}],\ for\ n = m.
\end{aligned}
$$

For given values of supply voltage Vs, operating frequency f, circuit element values C_1, L_o, C_o and R, we can find out I_1 and output current $Io(\theta)$ (by solving the matrix Eq. (2.55)). Once, output current is known shunt capacitor voltage $V_{c1}(\theta)$ can be obtained at any time instant from Eq. (2.45).

If the circuit is operating at optimum condition, the input power will be equal to the output power. For a given value of power output Po, input current I1(=Po/vs) can be assumed constant.

The flow-chart and program of the analysis assuming non-sinusoidal output current, is given in appendix (B).

2.5.3 Comparison of Analysis Results With Results of Simulation

For a given value of circuit elements, we can get solutions for circuit responses by the two methods of analysis, (1) analysis assuming output current sinusoidal, (2) analysis assuming non-sinusoidal output current. There are several other methods, using which the class-E circuit can be analysed exactly (e.g., time-domain analysis and s-domain analysis). The importance of harmonic analysis method is that it leads to an accurate design method for optimum operation of the class-E amplifier. We will take a set of circuit element values, with which the circuit operates at optimum operating conditions. The responses of the circuit will be compared with results of simulation and the results obtained by the two analysis methods described in previous sections. The circuit values are given below:

$$V_s = 100V,\ L_1 = 5mH,\ f = 50KHz,\ D = 0.5,\ R = 2.7484\Omega,$$
$$C_1 = 0.2292\mu F,\ C_o = 0.1317\mu F,\ L_o = 87.4843\mu H.$$

Fig. 2.28 shows the plot of three curves of V_{c1}. Curve 1 has been obtained by calculation of $V_{c1}(\theta)$ from π to 2π, using Eq. (2.13). This equation has been derived assuming output current I_o pure sinusoidal (i.e., assuming fundamental component only). This analysis gives approximate solution. The peak value of V_{c1} is 343.76V, and at $\theta = 2\pi$ its magnitude is 6.3V with negative slope. Curve 2 has been plotted by calculating $V_{c1}(\theta)$ from π to 2π, using Eq. (2.45). For this case, the output current I_o is assumed non-sinusoidal. While calculation, first ten harmonics were considered in the I_o. Since, magnitudes of harmonics beyond third harmonics, are very small. Hence, assuming ten harmonics in the calculation gives almost same result, as it is obtained by exact analysis shown by curve 3. The peak value of V_{c1} in curve 2 is 358.87V, and at $\theta = 2\pi$ its magnitude and slope is almost zero. Curve 3 and curve 2 are almost same. The peak value of V_{c1} in curve 3 is 359.45v, and at $\theta = 2\pi$ its magnitude and slope are almost zero.

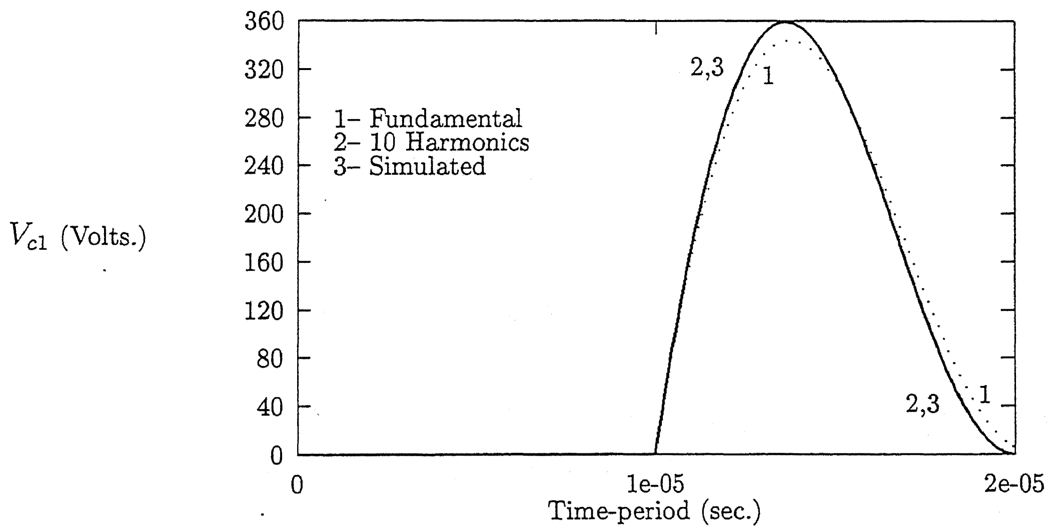

Figure 2.28: Plots of V_{c1} to compare with simulation result.

Fig. 2.29 is plot of I_1 and I_o. The input current I_1 has been assumed 20.0A in the analysis, and the plotted I_1 has been obtained by simulation. Three curves of I_o has been shown in the figure. Curve 1 has been obtained by calculating I_o from Eq. (2.8) i.e., assuming pure sinusoidal output current I_o. Its magnitude at $\theta = 2\pi$ is 21.83A, and peak values are -37.46A and +37.46A. Curve 2 is plot of output current which has been calculated from Eq. (2.40), and assuming first ten harmonics only. The analysis assuming output current with first ten harmonics gives almost the same result as obtained by exact analysis. This analysis gives the value of I_o 19.46A at $\theta = 2\pi$. Its peak values are -38.49A and +38.49A. Curve 3 shows the plot

of output current I_o obtained by simulation result (i.e., exact analysis of the circuit). At $\theta = 2\pi$ the magnitude of I_o is 19.86A. Its peak values are -36.33A and +40.02A.

The values of I_o at switch turn-off, given by curves 1, 2, and 3 are -22.31A, -19.98A and -22.63A. Current I_{sw1} (equal to $I_1 - I_o$) at the time of switch turn-off gives the magnitude of current which is required to be forced turned-off.

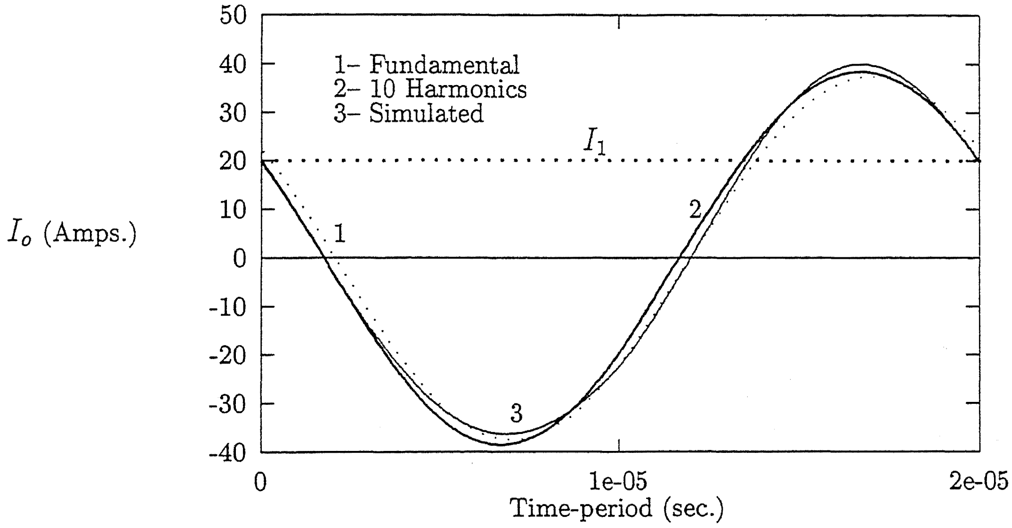

Figure 2.29: Plots of I_o to compare with simulation result.

2.6 Design Equations

2.6.1 Approximate Design Equations With Sinusoidal Output Current

The approximate design equations for class-E amplifier can be obtained for optimum operating condition. For unity efficiency, input power P_i is equal to output power P_o. Hence, for given power output and supply voltage,

$$R_{dc} = \frac{V_s^2}{P_o} \tag{2.58}$$

Where, R_{dc} is equivalent resistance of the amplifier as seen by the power supply. Hence,

$$R_{dc} = \frac{V_s}{I_1} \tag{2.59}$$

Using Eq. (2.31) and (2.59),

$$R_{dc} = \frac{1}{\pi w C_1} \tag{2.60}$$

Since, at optimum operating condition, efficiency is unity. Substituting this efficiency in Eq. (2.34), we get,

$$V_s = \frac{I_{om}^2 R}{2I_1} \tag{2.61}$$

Hence, substituting V_s from Eq. (2.61) in Eq. (2.59) and using Eq. (2.29)and (2.30), we get,

$$R = 0.5768 R_{dc} \tag{2.62}$$

For a given value of operating frequency f, supply voltage V_s and power output P_o, the values of load resistance R and shunt capacitance C_1 can be obtained using Eq. (2.58)–(2.62).

For a given quality-factor Q_l and operating frequency f, output inductance is given by,

$$L_o = \frac{Q_l R}{2\pi f} \tag{2.63}$$

Here, high chosen value of Q_l reduces harmonic content in output.

Using Eq. (2.21), (2.30), (2.60) and (2.62), we can obtain value of C_o, as given below,

$$C_o = \frac{1}{w^2 L_o}[1 + \frac{1.1534}{Q_l - 1.1534}] \tag{2.64}$$

The above design equations show that for optimum operation of class-E amplifier, there is a unique relation among circuit elements i.e., there is only one set of circuit elements are possible for a given frequency and Q_l, power output Po and supply voltage Vs.

2.6.2 Approximate Design Example

The design example is for an amplifier to deliver power 2 KW at 50 KHz, $V_s = 100V$, $L_1 = 5mH$ (RF CHOKE), D=$\frac{1}{2}$, and a given quality factor Q_l.

Using equations of previous section, we have obtained approximate values of circuit elements. These values are very near to the circuit elements which give optimum class-E characteristic. As we increase quality factor, the output current becomes more sinusoidal, hence, the fundamental component follow the actual waveform. Thus, the approximate values tend towards the optimum values with increase in quality factor. Table-2.3, shows approximate design values with four value of quality factor.

Table-2.3

Approximate Circuit Design Values

$Q_l = 5.0$	$Q_l = 10.0$	$Q_l = 20.0$	$Q_l = 40.0$
$R = 2.8840\Omega$	$R = 2.8840\Omega$	$R = 2.8840\Omega$	$R = 2.8840\Omega$
$C_1 = 0.2026\mu$F	$C_1 = 0.2026\mu$F	$C_1 = 0.2026\mu$F	$C_1 = 0.2026\mu$F
$C_o = 0.2869\mu$F	$C_o = 0.1247\mu$F	$C_o = 0.0586\mu$F	$C_o = 0.0284\mu$F
$L_o = 45.9003\mu$H	$L_o = 91.8007\mu$H	$L_o = 183.6014\mu$H	$L_o = 367.2028\mu$H

Effect of Quality Factor On Circuit Responses

The four design values shown above in table-2.3 have been simulated. Fig. 2.30 to Fig. 2.32 shows circuit responses V_{c1}, I_1 and I_o for one cycle, with different quality-factors. Curves are numbered 1 to 4. Curve 1 is for $Q_l = 5$, curve 2 is for $Q_l = 10$, curve 3 is for $Q_l = 20$ and curve 4 is for $Q_l = 40$. Fig. 2.30 shows that peak value of V_{c1} goes on decreasing with increasing Q_l, slope at $\theta = 2\pi$ also moves towards zero.

Fig. 2.31 is zoomed portion of Fig. 2.30 near $\theta = 2\pi$. It is observed that with increase in quality-factor, our circuit values tend to be optimum. Fig. 2.32 is plot of I_1 and I_o. It is observed that there is very little decrease in I_1 with increase in quality-factor. As Q_L is increased I_o at $\theta = 2\pi$ tends to equal to input current I_1. Phase shift in output current is observed with quality-factor.

2.6.3 Modified Procedure for More Accurate Design

From the harmonic analysis of output current I_o, it is evident that first three harmonics are significant. If in our design equations, three or more harmonic components are considered, we will get more accurate values of the circuit elements. In this procedure, we use the approximate designed circuit element values of previous section as starting point.

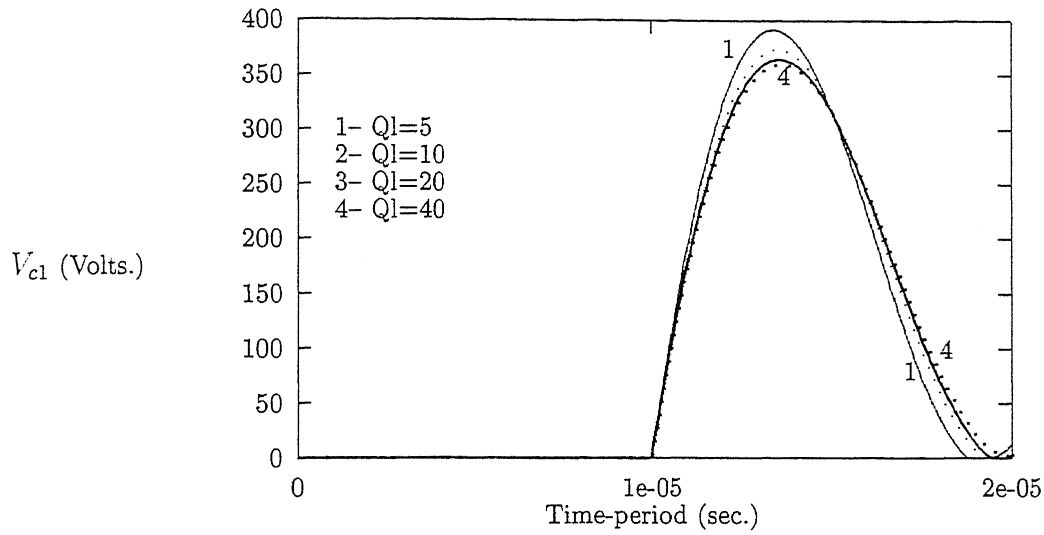

Figure 2.30: Plots of V_{c1} with different Q_l.

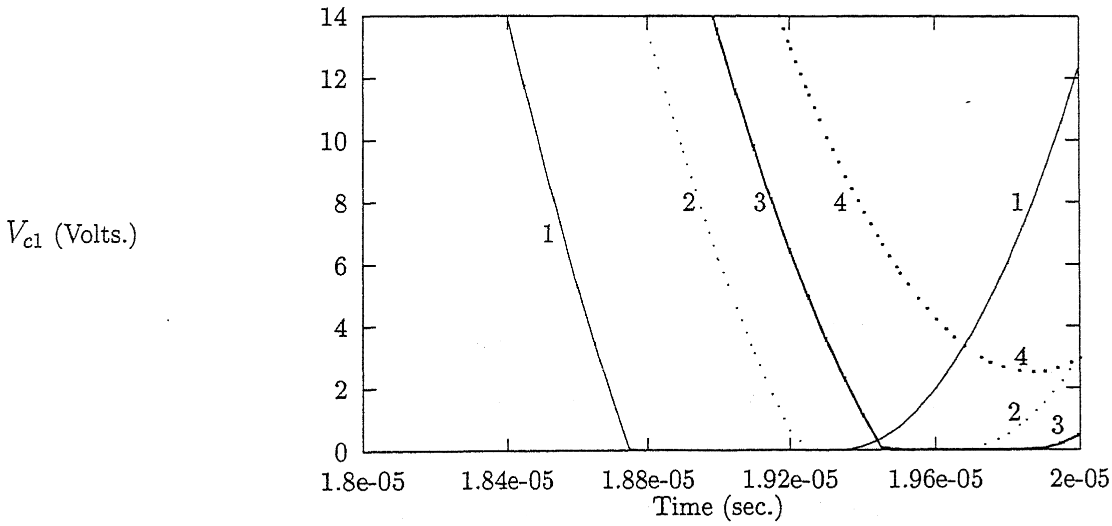

Figure 2.31: Magnified plot of V_{c1}, near $\theta = 2\pi$ with different Q_l.

We find harmonics of $V_{c1}(\theta)$, assuming only the fundamental current $Iom_1\angle\phi_1$. The $V_{c1}(\theta)$ from Eq. (2.13) can be written in terms of its components,

$$V_{c1}(\theta) = V_{c1_{av}} + Vc_n \tag{2.65}$$

$$Vc_n = \sum_{n=1}^{k}[a_n\cos(n\theta) + b_n\sin(n\theta)] \tag{2.66}$$

Where, $k \to \infty$. and

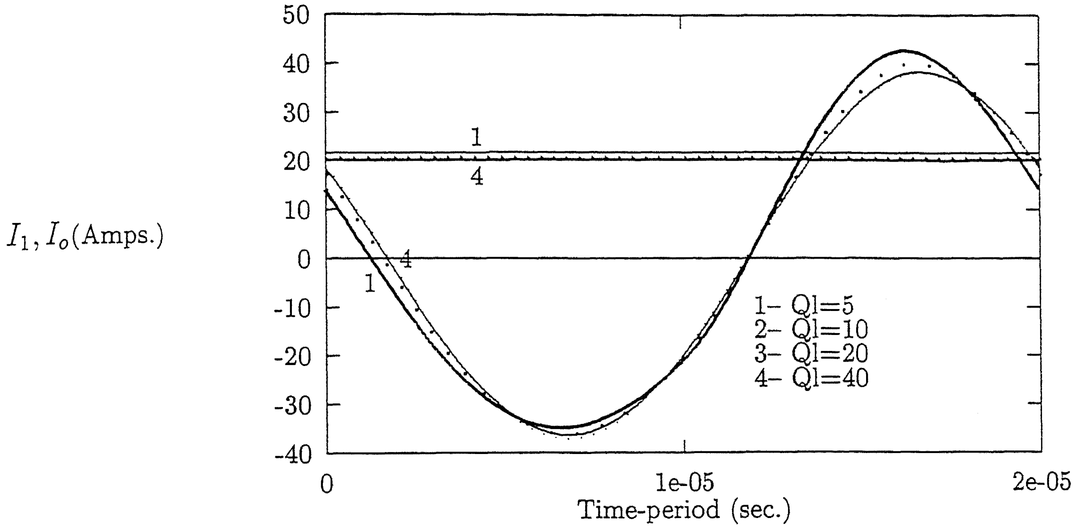

Figure 2.32: Plots of I_1 and I_o with different Q_l.

$$\begin{aligned} a_n &= \frac{1}{\pi w C_1}[\frac{1-(-1)^n}{n^2}I_1 + \frac{\pi}{2}Iom_1\cos\phi_1 \\ &+\frac{1+(-1)^n}{1-n^2}Iom_1\sin\phi_1] \end{aligned} \quad (2.67)$$

$$\begin{aligned} b_n &= \frac{1}{\pi w C_1}[-\frac{\pi}{n}I_1 - \frac{1-(-1)^n}{n}Iom_1\cos\phi_1 \\ &+\frac{n(1+(-1)^n)}{1-n^2}Iom_1\cos\phi_1 - \frac{\pi}{2}Iom_1\sin\phi_1] \end{aligned} \quad (2.68)$$

At steady-state, the n^{th} harmonic output current $\overline{I}o_n$, can be found by dividing n^{th} harmonic $\overline{V}c_n$ with n^{th} harmonic series impedance $\overline{Z}o_n$. Hence,

$$\begin{aligned} \overline{I}o_n &= \frac{\overline{V}c_n}{\overline{Z}o_n} \quad (2.69) \\ Where, & \\ \overline{Z}o_n &= \sqrt{X_n^2+R^2}\angle\arctan\frac{X_n}{R} \\ and, & \\ \overline{V}c_n &= \sqrt{a_n^2+b^2}\angle\arctan\frac{a_n}{b_n} \\ and, & \\ X_n &= nwL_o - \frac{1}{nwC_o} \end{aligned}$$

After obtaining. n—harmonic components of $\overline{I}o_n$, we find new value of fundamental current, by putting zero magnitude and zero slope conditions of shunt capacitor voltage $V_{c1}(\theta)$ at $\theta = 2\pi$. $V_{c1}(\theta)$ is given by Eq. (2.45), is used to satisfy the design conditions. Hence, putting $V_{c1}(\theta)$ equal to zero from Eq. (2.45), we obtain;

$$Iom_1 \cos\phi_1 = -\frac{1}{2}[\pi I_1 + \sum_{n=2}^{k} \frac{1-(-1)^n}{n} Iom_n \cos\phi_n] \tag{2.70}$$

Imposing another design condition $\frac{dV_{c1}(\theta)}{d\theta} = 0.0$ for optimum operating condition, we obtain;

$$Iom_1 \sin\phi_1 = I_1 - \sum_{n=2}^{k} Iom_n \sin\phi_n \tag{2.71}$$

With modified value of fundamental current, we obtain from Eq. (2.47);

$$C_1 = \frac{1}{2\pi w V_s}[\frac{\pi^2}{2} I_1 + \sum_{n=1}^{k} Iom_n\{\frac{1-(-1)^n}{n^2}\sin\phi_n - \frac{(-1)^n\pi}{n}\cos\phi_n\}] \tag{2.72}$$

Since, the value of C_1 has been modified, hence, the voltage $V_{c1}(\theta)$ will also be modified. New, n^{th} harmonic components of $V_{c1}(\theta)$, will depend on all harmonic components of output current (not only the fundamental !). Hence, Eq. (2.66) will be modified. Suppose modified coefficients are a_m and b_m for m^{th} harmonic of $V_{c1}(\theta)$, hence,

$$\begin{aligned} a_m &= \frac{1}{\pi w C_1}[\frac{\{1-(-1)^m\}}{m^2} I_1 + \frac{\pi}{2m} Iom_m \cos\phi_m \\ &+ \sum_{n=1, n\neq m}^{k} \frac{\{(-1)^{m+n} - 1\}}{m^2 - n^2} Iom_n \sin\phi_n] \end{aligned} \tag{2.73}$$

and,

$$\begin{aligned} b_m &= \frac{1}{\pi w C_1}[-\frac{\pi}{m} I_1 - \frac{\pi}{2m} Iom_m \sin\phi_m + \sum_{n=1, n\neq m}^{k} \frac{m\{(-1)^{m+n} - 1\}}{n(m^2 - n^2)} Iom_n \cos\phi_n \\ &+ \sum_{n=1}^{k} \frac{(-1)^n\{1-(-1)^m\}}{mn} Iom_n \cos\phi n] \end{aligned} \tag{2.74}$$

From above equations; we find,

$$\overline{V}c_n = \sqrt{a_n^2 + b_n^2}\angle\arctan\frac{a_n}{b_n} \tag{2.75}$$

Hence, new value of fundamental series impedance will be,

$$Zo_1 = \sqrt{\frac{a_1^2 + b_1^2}{Iom_1}} \tag{2.76}$$

Here onwards, we have two design options:

1. Design with V_s, P_o, Q_l and f constant.
2. Design with R, Q_l and f constant.

Design with V_s, P_o, Q_l and f constant

Since, the output current has been modified from Eq. (2.70) and (2.71); to keep output power P_o constant, the new value of output resistance R will be,

$$R = \frac{2P_o}{\sum_{m=1}^{k} Iom_m^2} \tag{2.77}$$

Hence, with fixed Q_l series inductor L_o is given as,

$$L_o = \frac{Q_l R}{w} \tag{2.78}$$

Hence, new value of series capacitor C_o is given by,

$$C_o = \frac{1}{w^2 L_o - w\sqrt{Zo_1^2 - R^2}} \tag{2.79}$$

a Here, we have new values of R, L_o, C_1 and C_o; now we can find new values of $\overline{Z}o_n$. With new values of $\overline{Z}o_n$ and $\overline{V}c_n$, we again find $\overline{I}o_n$ from Eq. (2.69), and repeat the whole procedure from Eq. (2.69) to (2.79), till the values of C_1, R, L_o and C_o are stablized (i.e., the values computed in two consequtive iteration are same).

Using this procedure, we have computed four designed values,shown in table-2.4. The values are with considering only fundamental, only first

two, only first three and only first ten harmonics of output current. It is evident that as we consider more harmonics, the designed values become more accurate, but consideration above three harmonics does not contribute significant change in the design values.

Table-2.4			
Circuit Design Values With More Accurate Procedure			
Design With V_s, P_o, Q_l and f constant			
Considering N Number of Harmonics			
N=1	N=2	N=3	$N = 10$
$R = 2.8840\Omega$ $C_1 = 0.2026\mu$F $C_o = 0.1247\mu$F $L_o = 91.8007\mu$H	$R = 2.7708\Omega$ $C_1 = 0.2275\mu$F $C_o = 0.1305\mu$F $L_o = 88.1976\mu$H	$R = 2.7547\Omega$ $C_1 = 0.2280\mu$F $C_o = 0.1314\mu$F $L_o = 87.6858\mu$H	$R = 2.7484\Omega$ $C_1 = 0.2292\mu$F $C_o = 0.1317\mu$F $L_o = 87.4843\mu$H

Design with R, Q_l and f constant

In this case, we have fixed R, Q_l (i.e., also L_o) and f. In the design procedure, the output current $\overline{I}o_n$ is changing to satisfy the design conditions of Eq. (2.70) and (2.71). Hence, with constant R, the output power P_o will not be constant. The R is hold fixed, at the value designed by approxmate design method. At optimum operating condition, the input power should be equal to the output power, because of unity efficiency. To satisfy this condition, we will have to vary the input supply voltage V_s. (Chang in Vs is very small.)

After Eq. (2.76), we apply following procedure. The new output power is given by,

$$P_o = \frac{\sum_{m=1}^{k} Iom_m^2 R}{2} \tag{2.80}$$

New supply voltage V_s to make input power equal to output power,

$$V_s = \frac{P_o}{I_1} \tag{2.81}$$

Also, new value of series capacitor C_o is given by,

$$C_o = \frac{1}{w^2 L_o - w\sqrt{Zo_1^2 - R^2}} \tag{2.82}$$

Here, we obtained new values of P_o, V_s, C_o and $\overline{V}c_n$. Now, we calculate new values of $\overline{Z}o_n$ and find $\overline{I}o_n$ from Eq. (2.69). We repeat the procedure

from Eq. (2.69) to (2.76) and (2.80) to (2.82), till the values of C_1 and C_o are stablized.

Using this procedure, we have computed four designed values shown in table-2.5, with N=1, N=2, N=3 and N=10. It is evident that only C_1 and C_o has been modified. The modified values for $N > 3$ makes the circuit to operate in optimum operating condition.

Table-2.5 Circuit Design Values With More Accurate Procedure Design With R, Q_l and f constant Considering N Number of Harmonics			
N=1	N=2	N=3	$N = 10$
$R = 2.8840\Omega$	$R = 2.8840\Omega$	$R = 2.8840\Omega$	$R = 2.8840\Omega$
$C_1 = 0.2026\mu$F	$C_1 = 0.2186\mu$F	$C_1 = 0.2178\mu$F	$C_1 = 0.2184\mu$F
$C_o = 0.1247\mu$F	$C_o = 0.1254\mu$F	$C_o = 0.1255\mu$F	$C_o = 0.1256\mu$F
$L_o = 91.8007\mu$H	$L_o = 91.8007\mu$H	$L_o = 91.8007\mu$H	$L_o = 91.8007\mu$H

The flow-chart and computer program of the modified procedure with more accurate design is given in appendix (C).

Circuit Responses With More Accurate Design Values

Fig. 2.33 to 2.36 shows plots of circuit responses V_{c1}, I_1 and I_o with four designed circuit values, as it is shown in table-2.4. From Fig. 2.33 curve no.1 is with design considering fundamental harmonic only, curve no.2 is with design considering fist two harmonics, curve no.3 is with design considering first three harmonics and curve no.4 is with design considering first ten harmonics. From the figures, it is evident that only first two harmonics are significant. Only curve no.1 is distinct, rest of the curves are almost covered by curve no.10. As we include more and more harmonic components the design becomes more and more accurate. Fig. 2.34 shows the zoomed portion of Fig. 2.33 near $\theta = 2\pi$. In Fig. 2.34 curves are distinct, and it can be observed that as we include more number of harmonics, we reach the zero magnitude and zero slope condition of V_{c1}. Fig. 2.35 shows that output current I_o at $\theta = 2\pi$ tends to equalize the input current I_1 in magnitude, as we consider more and more number of harmonics. Fig. 2.36 is zoomed plot of curve shown in Fig. 2.35 near $\theta = 0.0$. It is clearly observed the movement of output current I_o towards input current I_1 at $\theta = 0.0$. There is very small decrease in input current I_1 as we consider more number of harmonics. This is because, since the output power is constant, the input power will be higher than the output power to meet the losses at turn-on.

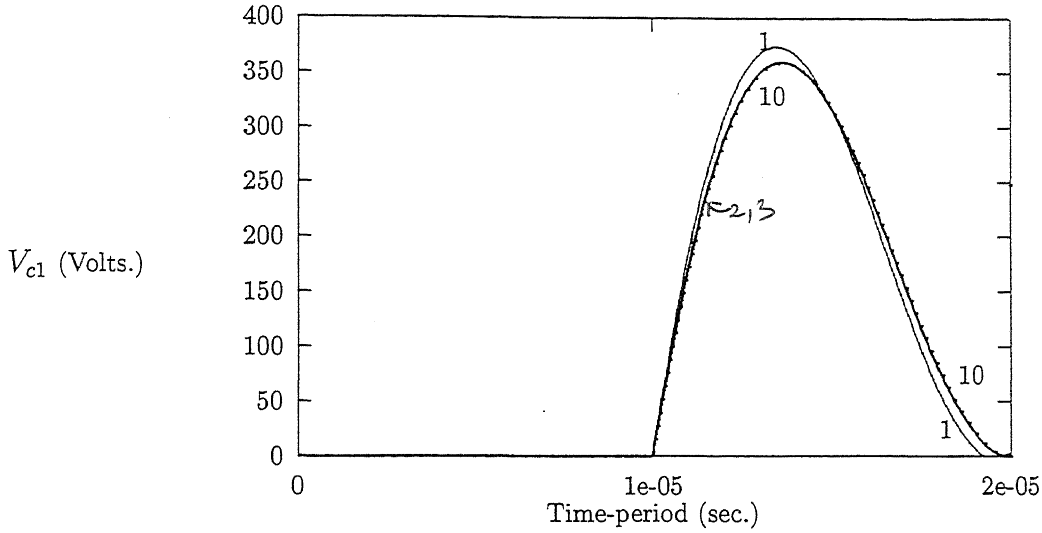

Figure 2.33: Plots of V_{c1} with different harmonics in design (P_o constant).

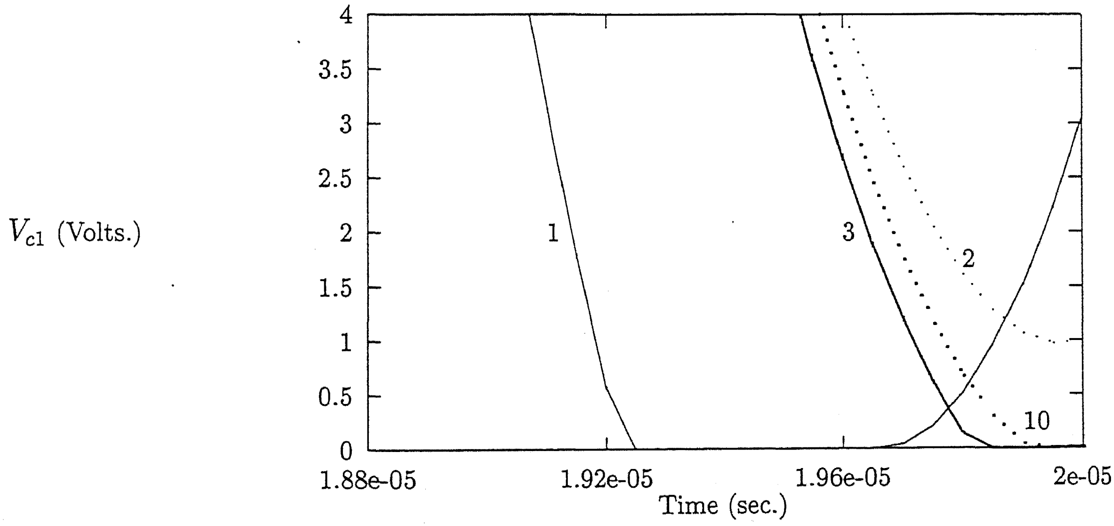

Figure 2.34: Magnified plots of V_{c1}, near $\theta = 2\pi$ with different harmonics in design (P_o constant).

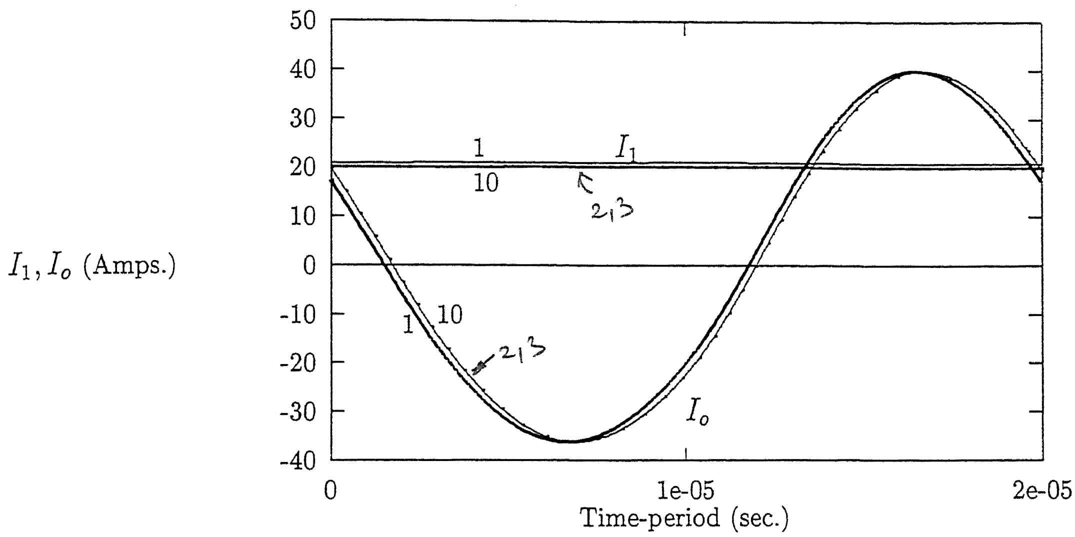

Figure 2.35: Plot of I_1 and I_o with different harmonics in design (P_o constant).

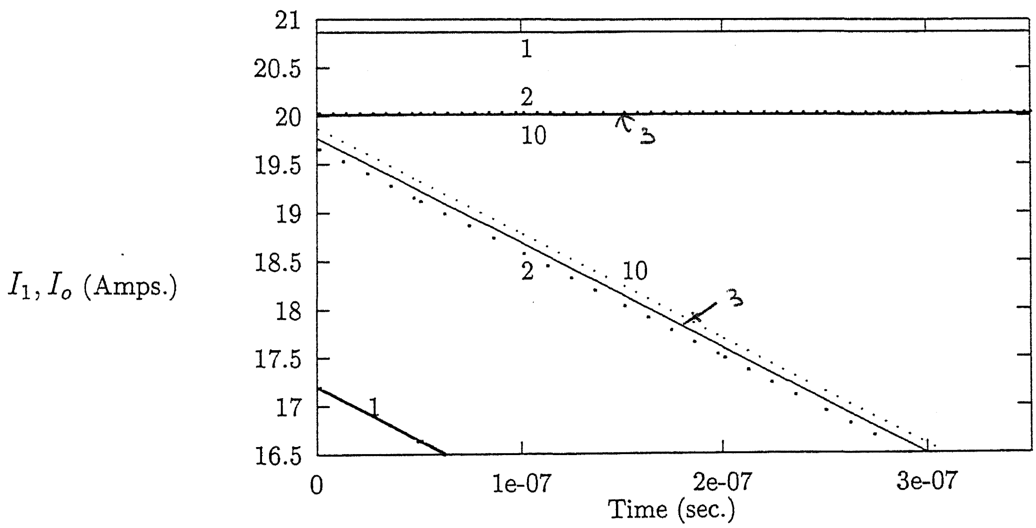

Figure 2.36: Magnified plots of I_1 and I_o near $\theta = 0.0$ with different harmonics in design (P_o constant).

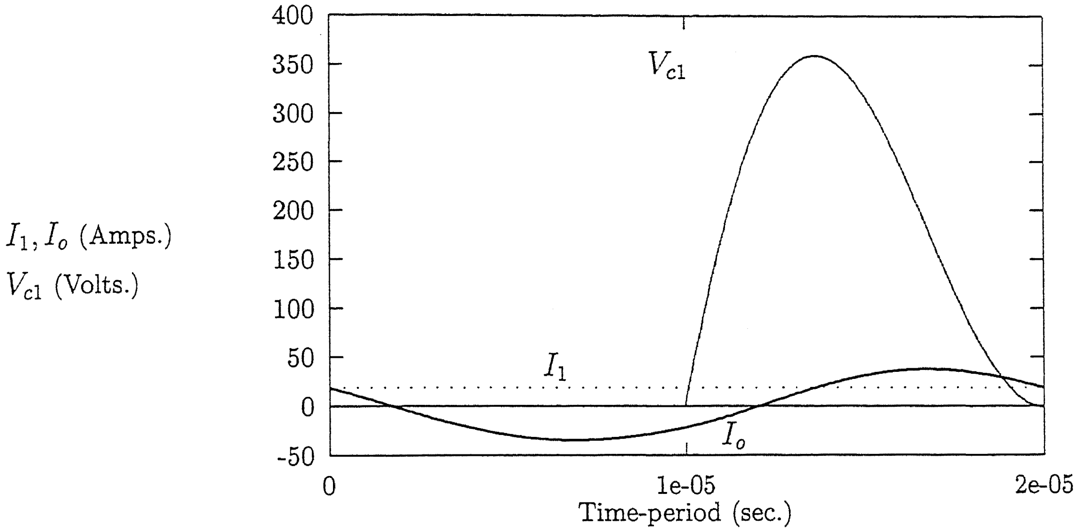

Figure 2.37: Optimum circuit response with design assuming R, Q_l and f constant.

Fig. 2.37 is the plot of V_{c1}, I_1 and I_o with circuit values given in table-2.5 for N=10. The circuit responses show that the operating condition is optimum.

Chapter 3

Analysis and Design of Class-E Push-Pull Configuration

3.1 Introduction

In the basic single ended class-E amplifier analysed in chapter -2, we have observed that the load current and hence load voltage for periods zero to DT and DT to T were not symmetrical, even in steady state. This is because, during these two intervals the circuit configurations were different. Therefore, this single ended class-E may contain even harmonic components in the output. For the application as an RF amplifier a second harmonic filter is required for this configuration.

Class-E push-pull amplifier provides a symmetrical configuration as shown in Fig. 3.1. It consists of two switches S1 and S2 and two shunt capacitors C_1 and C_2. The C_1 and C_2 may include any capacitance inherent in switch S1, diode D1 and switch S2, diode D2 respectively. Series tuned network consists of L_o, C_o and load resistance R. L_1 and L_2 are high value inductances (RF Chokes) in series with voltage sources to realize current sources.

A typical waveforms of push-pull class-E amplifier is shown in Fig. 3.2. The voltage magnitude and its slope is always zero across the switches at the time of turn-on. At optimum operation, the voltage across S1 and S2 are V_{c1} and V_{c2} respectively, in switch-off duration. The output current I_o is almost sinusoidal.

The class-E push-pull amplifier can be operated with two or three states as shown in Fig. 3.3 and Fig. 3.4. With two modes of operation the duty

cycle is 50-percent. For three modes of operation the duty cycle is taken greater than 50-percent.

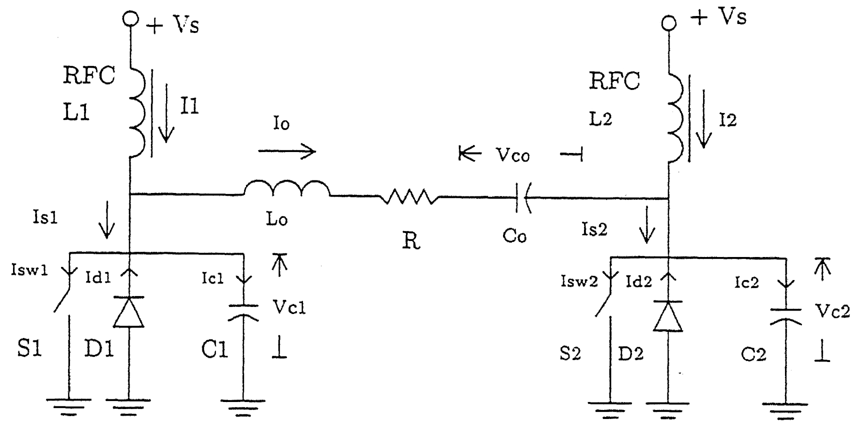

Figure 3.1: The push-pull class-E amplifier.

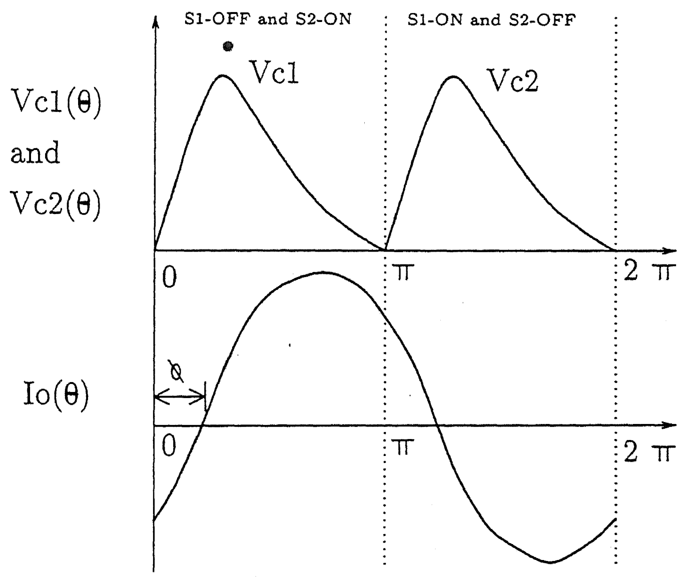

Figure 3.2: Typical sketch of push-pull class-E waveforms Vc1, Vc2 and Io at steady-state.

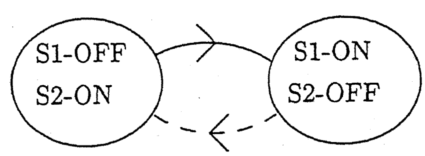

Figure 3.3: Two modes of push-pull class-E amplifier operation.

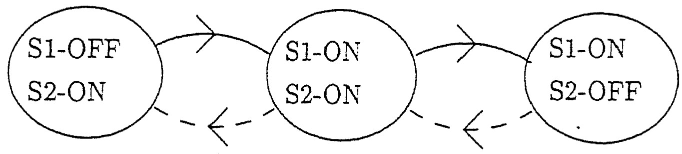

Figure 3.4: Three modes of push-pull class-E amplifier operation.

3.2 Exact Analysis of Push-Pull Class-E Amplifier With Two Modes of Operation (Using State-Equations)

The class-E push-pull circuit to be analyzed is shown in Fig. 3.1. The present circuit have six energy storage elements, hence, there are six variables. The variables I_1, I_2, I_o, V_{c1}, V_{c2} and V_{co} are associated with L_1, L_2, L_o, C_1, C_2 and C_o respectively. To maintain symmetry in the circuit , the values of C_1 and C_2, L_1 and L_2 should be same. Also ratings of switches S1 and S2, diodes D1 and D2 should be equal. Circuit equations have been written for two states of operation.

Circuit Equations for Mode-1

The equivalent circuit diagram for the circuit mode-1 S1 "off" and S2 "on" is shown in Fig. 3.5. Since S2 "on" is associated with diode in antiparallel (which conducts for reverse current), it is assumed as a bidirectional switch during "on" period. The V_{c2} is zero for this interval. There are five first order differential equations for this state.

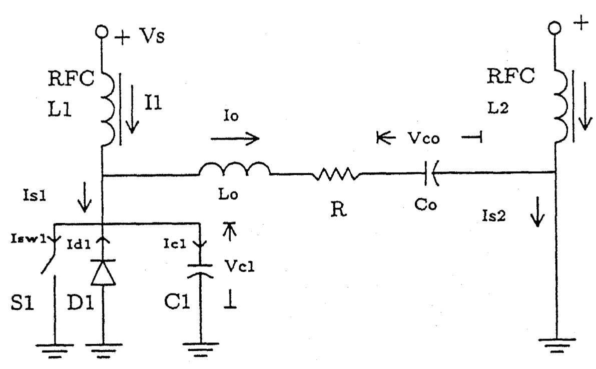

Figure 3.5: Equivalent circuit for mode-1.

$$\frac{dI_1}{dt} = \frac{V_s - V_{c1}}{L_1} \tag{3.1}$$

$$\frac{dV_{c1}}{dt} = \frac{I_1 - I_o}{C_1} \tag{3.2}$$

$$\frac{dI_2}{dt} = \frac{V_s}{L_2} \tag{3.3}$$

$$\frac{dI_o}{dt} = \frac{V_{c1} - V_{co}}{L_o} - \frac{R}{L_o} I_o \tag{3.4}$$

$$\frac{dV_{co}}{dt} = \frac{I_o}{C_o} \tag{3.5}$$

Circuit Equations for Mode-2

For mode-2 the switch state is S1 "on" and S2 "off" as shown in Fig. 3.6. The five first order differential equations have been written. For this state V_{c1} is zero.

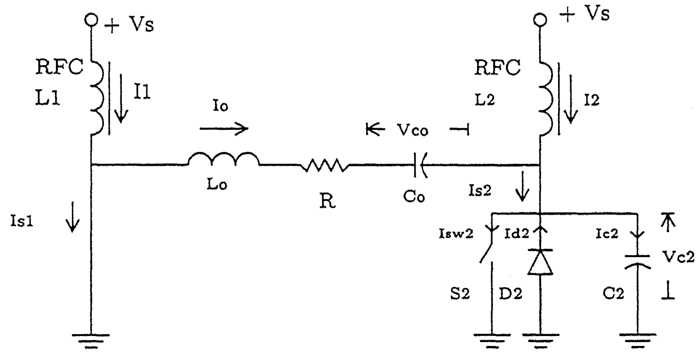

Figure 3.6: Equivalent circuit for mode-2.

$$\frac{dI_1}{dt} = \frac{V_s}{L_1} \tag{3.6}$$

$$\frac{dI_2}{dt} = \frac{V_s - V_{c2}}{L_2} \tag{3.7}$$

$$\frac{dV_{c2}}{dt} = \frac{I_2 + I_o}{C_2} \tag{3.8}$$

$$\frac{dI_o}{dt} = -\frac{V_{c2} + V_{co}}{L_o} - \frac{R}{L_o} I_o \tag{3.9}$$

$$\frac{dV_{co}}{dt} = \frac{I_o}{C_o} \tag{3.10}$$

Following optimum circuit design values have been taken.

$$L1 = L2 = 5.0mH; C1 = C2 = 0.20264\mu F; Co = 0.062374\mu F$$
$$Lo = 183.601\mu H; R = 5.768\Omega; Vs = 100.0V; f = 50KHz; D = 0.5$$

The simultaneous differential equations for the two states have been solved with above circuit values, and the steady-state values of parameters $(I_1, I_2, I_o, V_{c1}, V_{c2}, V_{co})$ have been shown in Fig. 3.7 to Fig. 3.10. The flow-chart and computer program for solution of the exact analysis with "two modes of operation" are given in Appendix-(D).

Fig. 3.7 is plot of V_{c1} and V_{c2} for one time-period at steady-state. These curves satisfy the optimum operating conditions of the push-pull amplifier. The peak value of V_{c1} and V_{c2} is 358.25V.

Fig. 3.8 is curve for V_{co} vs time-period. The first half of time-period is for S1 "off" and S2 "on" and the second half is for S2 "off" and S1 "on". The curve is almost sinusoidal and peak magnitude of voltage is 1904V.

Fig. 3.9 is curve of I_1, I_2, I_{s1}, I_{s2} for one time-period. since, source inductors L_1 and L_2 and voltage source value V_s are equal at the two ends, we have $I_1 = I_2 = 20.1A$. The current source values are almost constant. I_{s1} and I_{s2} are same in magnitude, but 180-degree out of phase. The peak switch current I_{s1} and I_{s2} is 57.4A. For optimum operating condition, when switch is off, the current I_{s1} and I_{s2} flow into capacitors C_1 and C_2, respectively. At the time of switch "off" the magnitude of switch current $I_{s_{off}}$ is 40.01A.

Fig. 3.10 is plot of output current I_o for one cycle of operation. It is almost sinusoidal. The peak magnitude of I_o is 37.3A.

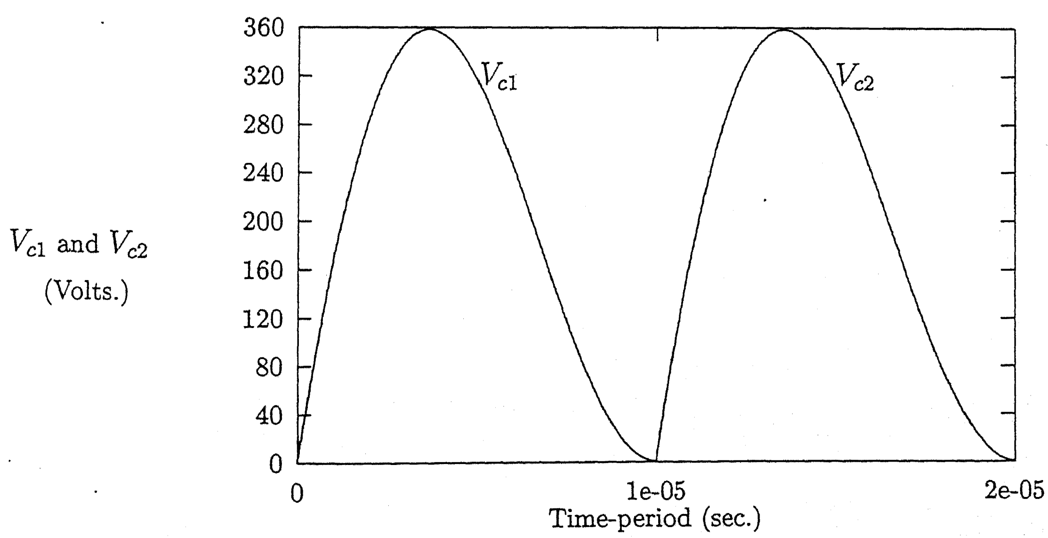

Figure 3.7: Curves of V_{c1} and V_{c2} for one cycle at steady-state.

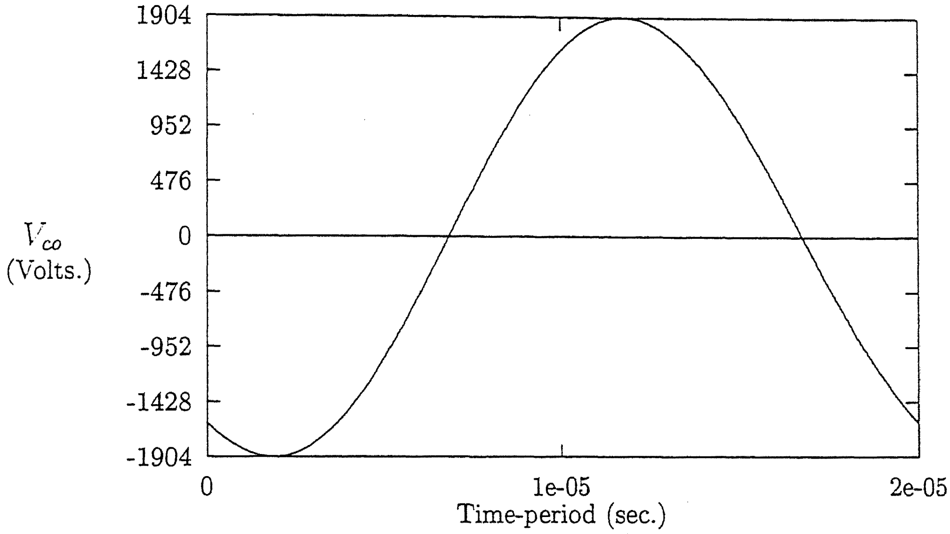

Figure 3.8: Curve of V_{co} for one cycle at steady-state.

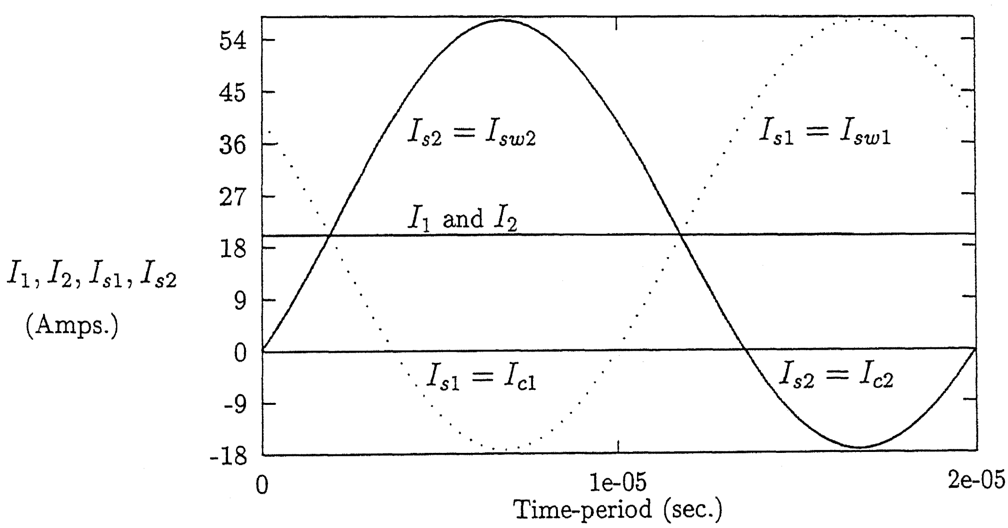

Figure 3.9: Curves of I_1, I_2, I_{s1} and I_{s2} for one cycle at steady-state.

Realizing Current Source as Voltage Source in Series With a Large Inductor

For analysis of push-pull class-E amplifier, the voltage source in series with a large inductor (RF Choke) can be replaced by a current source as shown in Fig. 3.11. The current source magnitude I_1 is decided by the magnitudes of voltage source and effective impedance in series with it. The power input to the amplifier is decided by the magnitudes of I_1 and I_2. The replacement of voltage source and large series inductor by a current source, makes the analysis simple, because the number of state equations are reduced.

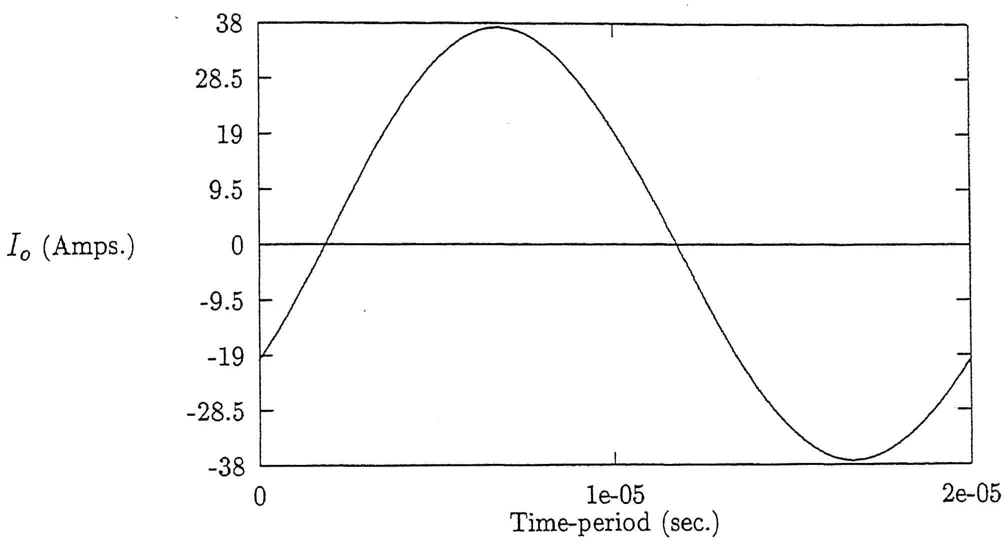

Figure 3.10: Curve of I_o for one cycle at steady-state.

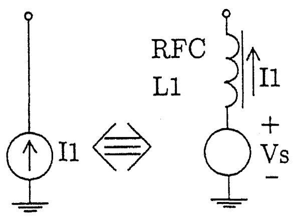

Figure 3.11: Current source realized by voltage source in series with a large inductor.

Voltage source in series with large inductor was replaced by the current source in Fig. 3.1, and the circuit was simulated. It gave the same results as shown from Fig. 3.7—3.10 for the same values of input currents and circuit elements.

3.3 Harmonic Analysis of Output Current

The push-pull class-E amplifier may contain harmonics in its output current wave. Because of symmetry, the even harmonics will be absent. Using "Discrete Fourier Transform Method", the harmonic components of output current I_o shown in Fig. 3.7, have been obtained. The peak magnitudes and phase angles are tabulated in Table-3.1. The harmonic components were calculated using NAG subroutine C06EAF.

Table-3.1		
Harmonics and Phases of Output Current		
Harmonic	Peak magnitude (Amp.)	Phase angle (Degree)
1	37.105	-31.996
3	0.2904	183.261
5	0.05544	181.236
7	0.01891	176.874
9	0.008546	168.236
11	0.004814	154.744
13	0.003382	139.128

From above table, it is evident that magnitudes of third and above harmonic components are negligible in comparison to fundamental component. Hence, the output current waveform is almost sinusoidal.

3.4 Analysis Assuming Constant Input Current and Sinusoidal Output Current With Two Modes of Operation

3.4.1 Analysis

For approximate analysis and design, the equivalent circuit of class-E push-pull amplifier is shown in Fig. 3.12. To analyze the circuit following assumptions have been made.

- Current through the load $I_o(\theta)$ is pure sinusoidal.
- Switches S1, S2 and diodes D1, D2 are ideal.
- Capacitors C_1 and C_2 may include any capacitances inherent in S1 and S2 respectively and are independent of any voltage across the switches.
- Inductors L_1 and L_2 are large enough to make currents I_1 and I_2 almost constant.
- L_1 and L_2 are equal. Switches S1 and S2, diodes D1 and D2 are of same ratings for symmetrical operation of the circuit.

For optimum operation of class-E push-pull amplifier the switch operating frequency will be different from the resonant frequency of L_o and C_o.

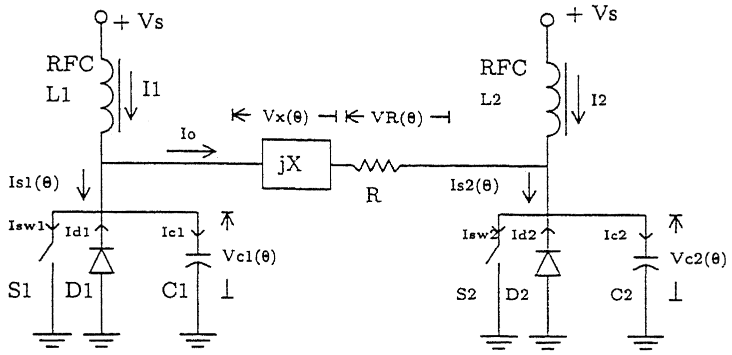

Figure 3.12: Equivalent circuit of push-pull class-E amplifier for approximate analysis.

Another equivalent circuit shows, series tuned load circuit consisting of jX, R and a voltage source V_c, as shown in Fig.3.13. The reactance X is equal to $(wL_o - \frac{1}{wC_o})$. Resistance R includes any series resistance inherent in L_o, C_o and equivalent 'lumped' a.c. resistances of switches and diodes. The voltage source $V_c(\theta)$ is equal to $V_{c1}(\theta)$ from 0 to π and is equal to $-V_{c2}(\theta)$ from π to 2π. The waveforms of $V_{c1}(\theta)$ and $V_{c2}(\theta)$ are shown in Fig. 3.2. The voltage source $V_c(\theta)$ is alternating, hence, output current I_o is also alternating.

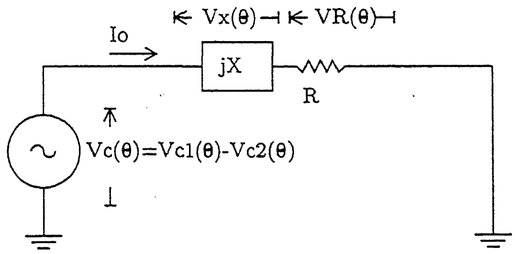

Figure 3.13: Equivalent circuit of push-pull class-E amplifier for approximate analysis.

For the case of S1 "off" and S2 "on", the equivalent circuit given in Fig. 3.13 will be applicable form 0 to π, and for case of S1 "on" and S2 "off", the equivalent circuit will be applicable from π to 2π. The output current and voltage are assumed sinusoidal. Thus,

$$I_o(\theta) = I_{om}\sin(\theta + \phi) \tag{3.11}$$

$$V_o(\theta) = I_{om}R\sin(\theta + \phi) \tag{3.12}$$

Where, θ(=wt) is "angular time", peak magnitude I_{om} and phase angle is ϕ (as shown in Fig. 3.2), are to be determined. $V_c(\theta)$ is sum of the voltages $V_x(\theta)$ and $V_R(\theta)$. Hence,

$$V_c(\theta) = I_{om}\sqrt{(R^2 + X^2)}\sin(\theta + \phi + \psi) \tag{3.13}$$

Where, $\psi = \arctan\left(\frac{X}{R}\right)$.

$V_c(\theta)$ is represented by following equation;

$$V_c(\theta) = V_{c1}(\theta) - V_{c2}(\theta) \tag{3.14}$$

Where, $V_{c1}(\theta)$ and $V_{c2}(\theta)$ can be obtained by following method.

For the duration S1 "off" and S2 "on" $V_{c2}(\theta)$ is zero, and

$$V_{c1}(\theta) = \frac{1}{wC_1}\int_0^\theta [I_1 - I_{om}\sin(u + \phi)]du \tag{3.15}$$

Hence,

$$V_{c1}(\theta) = \frac{1}{wC_1}[I_1\theta + I_{om}\cos(\theta + \phi) - I_{om}\cos\phi] \tag{3.16}$$

Where, θ is between 0 and π for D=0.5.

For the duration S1 "on" and S2 "off" the $V_{c1}(\theta)$ is zero, and

$$V_{c2}(\theta) = \frac{1}{wC_2}\int_\pi^\theta [I_2 + I_{om}\sin(u + \phi)]du \tag{3.17}$$

Hence,

$$V_{c2}(\theta) = \frac{1}{wC_2}[I_2(\theta - \pi) - I_{om}\cos(\theta + \phi) - I_{om}\cos\phi] \tag{3.18}$$

Where, θ is from π to 2π for D=0.5.

The average voltage across inductor L_1 or L_2 is zero. Hence the voltage across the source is equal to the average voltage across capacitors C_1 or C_2. Therefore, source voltage,

$$V_s = \frac{1}{2\pi}\int_0^{2\pi} V_{c1}(\theta)d\theta \tag{3.19}$$

From equations (3.16) and (3.19), we have,

$$V_s = \frac{1}{2\pi wC_1}[\frac{\pi^2}{2}I_1 - 2I_{om}\sin\phi - \pi I_{om}\cos\phi] \tag{3.20}$$

Also, the source voltage,

$$V_s = \frac{1}{2\pi}\int_0^{2\pi} V_{c2}(\theta)d\theta \tag{3.21}$$

From equations (3.18) and (3.21), we have,

$$V_s = \frac{1}{2\pi w C_2}[\frac{\pi^2}{2}I_2 - 2I_{om}\sin\phi - \pi I_{om}\cos\phi] \tag{3.22}$$

From equations (3.20) and (3.22), it is evident that for symmetry, we should have equal voltage sources V_s, equal shunt capacitors C_1 and C_2, equal RF Choke inductors L_1 and L_2. The voltage across the series impedance Z_o, is $V_c(\theta)$. Where, $V_c(\theta)$ is given by Eq. (3.14). Fundamental component of $V_c(\theta)$ is obtained as,

$$V_{c_{fundamental}}(\theta) = \frac{1}{\pi w C_1}[(-4I_1+\pi I_{om}\cos\phi)\cos\theta+(2\pi I_1-\pi I_{om}\sin\phi-4I_{om}\cos\phi)]\sin\theta \tag{3.23}$$

In above equation, I_2 and C_2 have been replaced by I_1 and C_1 respectively. From Eq. (3.13), $V_c(\theta)$ can be written as,

$$V_c(\theta) = [RI_{om}\sin\phi + XI_{om}\cos\phi]\cos\theta + [RI_{om}\cos\phi - XI_{om}\sin\phi]\sin\theta \tag{3.24}$$

Equating coefficients of $\sin\theta$ *and* $\cos\theta$ from Eq.(3.23) and (3.24), we get following equations.

$$-4I_1 - \pi w C_1 R I_{om}\sin\phi + \pi(1 - wC_1X)I_{om}\cos\phi = 0 \tag{3.25}$$

and,

$$2\pi I_1 + \pi(-1 + wC_1X)I_{om}\sin\phi - (4 + \pi w C_1 R)I_{om}\cos\phi = 0 \tag{3.26}$$

Using Eq. (3.25), (3.26) and (3.20), we get following matrix equation;

$$\begin{bmatrix} -4 & -\pi w C_1 R & (\pi - \pi w C_1 X) \\ 2\pi & (\pi w C_1 X - \pi) & (-4 - \pi w C_1 R) \\ \frac{\pi^2}{2} & -2 & -\pi \end{bmatrix} \begin{bmatrix} I_1 \\ I_{om}\sin\phi \\ I_{om}\cos\phi \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2\pi w C_1 V_s \end{bmatrix}$$

From above matrix equation, we can solve for I_1, I_{om} and ϕ, for a given set of circuit element values. The angle ϕ is obtained as,

$$\phi = \arctan\left[\frac{-R - \frac{\pi}{2}X + \frac{\pi}{2wC_1} - \frac{4}{\pi wC_1}}{-[X - \frac{\pi}{2}R - \frac{1}{wC_1}]}\right] \tag{3.27}$$

Analysis For Optimum Operation

Since, all circuit elements are ideal, there are no losses. The loss occurs only if the switch-on take place at finite V_{c1} or V_{c2}. The requirement for a hundred-percent efficiency can be realized by setting equation (3.16) and (3.18) equal to zero at $\theta = \pi$ and $\theta = 2\pi$ respectively. Therefore,

$$V_{c1}(\theta)/_{\theta=\pi} = 0 = I_1\theta + I_{om}\cos(\theta + \phi) - I_{om}\cos\phi \tag{3.28}$$

Which gives,

$$\cos\phi = \frac{\pi I_1}{2I_{om}} \tag{3.29}$$

The slope of $V_{c1}(\theta)$ at the time of turn "on" of S1 must also be zero for optimum operation. Hence,

$$\frac{dV_{c1}(\theta)}{dt}/_{\theta=\pi} = 0 = I_1 - I_{om}\sin(\theta + \phi) \tag{3.30}$$

Which gives,

$$\sin\phi = -\frac{I_1}{I_{om}} \tag{3.31}$$

From Eq. (3.29) and (3.31), we get,

$$\phi = -32.481637^\circ = -0.5669115^c \tag{3.32}$$

From Eqs. (3.20), (3.29) and (3.31), we get

$$I_1 = V_s \pi w C_1 \tag{3.33}$$

Similarly, because of symmetry of the circuit following can also be obtained.

$$I_2 = V_s \pi w C_2 \tag{3.34}$$

From Eq. (3.31) and (3.32).

$$I_{om} = 1.8621 I_1 \tag{3.35}$$

Peak switch current is given by,

$$I_{s1pk} = I_1 + I_{om} \tag{3.36}$$

From Eq. (3.35) and (3.36), we get

$$I_{s1pk} = 2.8621 I_1 \tag{3.37}$$

For the value of θ_{vmax} at which the value of $V_{c1}(\theta)$ is maximum, from Eq. (3.16);

$$\frac{dV_{c1}(\theta)}{dt}/_{\theta=\theta_{vmax}} = 0 = I_1 - I_{om}\sin(\theta_{vmax} + \phi) \tag{3.38}$$

Using Eq. (3.31), (3.32) and (3.38), we get

$$\theta_{vmax} = 64.9633^\circ = 1.133823^c \tag{3.39}$$

Substituting value of θ_{vmax} in place of θ in Eq. (3.16) and using Eq. (3.31), (3.32) and (3.33), we obtain,

$$V_{c1_{max}} = 3.562 V_s \tag{3.40}$$

Similarly, we can also obtain,

$$V_{c2_{max}} = 3.562 V_s \tag{3.41}$$

Output power is given by

$$P_o = \frac{I_{om}^2 R}{2} \tag{3.42}$$

Input power is,

$$P_i = V_s(I_1 + I_2) \tag{3.43}$$

Efficiency of the amplifier is,

$$\eta = \frac{I_{om}^2 R}{2V_s(I_1 + I_2)} \tag{3.44}$$

3.4.2 Comparison of Approximate Analysis Results with Results of Simulations

The results obtained by above analysis will be approximate because the output current is assumed sinusoidal and input current is assumed constant. For a given value of $V_s = 100V, f = 50KHz$ and $C_1 = C_2 = 0.20264\mu F$, we calculate I_1 and I_2 20A from Eq. (3.33) and (3.34). The simulation result in Fig. 3.9 shows its value 20.1A.

The peak output current I_{om} given by Eq. (3.35) is calculated as 37.242A. The simulation result in Fig. 3.10 shows 37.3A.

Peak switch current I_{s1pk} is calculated as 57.242A from Eq. (3.37) and simulation result gives 57.4A, as in Fig. 3.9.

The values of V_{c1max} and V_{c2max} are calculated as 356.2V from Eq. (3.40) and (3.41). The simulation result gives $V_{c1max} = V_{c2max} = 358.25V$ from Fig. 3.7.

3.5 Design of Push-Pull Class-E Amplifier For Optimum Operation at D=0.5

3.5.1 Design Equations

For symmetrical operation of circuit, we have to keep $C_1 = C_2$. The supply voltage is common to both ends. Also, L_1 and L_2 are kept equal. From Eq. (3.33) and (3.34), it is observed that $Rdc1(= \frac{V_s}{I_1} = \frac{1}{\pi w C_1})$ and $Rdc2(= \frac{V_s}{I_2} = \frac{1}{\pi w C_2})$ are the resitive loads that the amplifier shows to respective power supplies at the two ends(Fig. 3.1). For Rdc1=Rdc2=Rdc and unity efficiency, we have,

$$Rdc = \frac{2V_s^2}{P_o} \tag{3.45}$$

Where, P_o is equal to sum of input powers at both end of the circuit at optimum operating condition. From Eq. (3.44) with $I_1 = I_2$ and $\eta = 1$, we get,

$$V_s = \frac{I_{om}^2 R}{4I_1} \tag{3.46}$$

Hence, substituting V_s from Eq. (3.33) and using Eq. (3.31) and (3.32) (also $Rdc = \frac{1}{\pi w C_1}$), we have,

$$R = 1.1536Rdc \tag{3.47}$$

Using Eq. (3.47) and $Rdc = \frac{1}{\pi w C_1}$, we have,

$$C_1 = \frac{0.0584421}{fR} \tag{3.48}$$

For a given quality factor Q_l,

$$L_o = \frac{Q_l R}{2\pi f} \tag{3.49}$$

Substituting calculated values of R, C_1, f, L_o (from above equations) and ϕ (from Eq. (3.32)) in Eq. (3.27), we obtain following equation for C_o;

$$C_o = \frac{1}{w^2 L_o}[1 + \frac{1.1524959}{Q_l - 1.1524959}] \tag{3.50}$$

Using Eq. (3.45)—(3.50) the circuit elements can be designed for optimum operating condition. The value of L_1 and L_2 should be chosen as large as possible to maintain input current almost constant.

3.5.2 Design Example

Same design problem has been taken as it was in chapter-2, with double amount of power. The amplifier is to deliver 4KW at 50 KHz,D=0.5, $V_s = 100V$, $L_1 = L_2 = 5.0mH$ (RF Choke), and $Q_l = 10$.

Using design equations of previous section the designed values obtained are given below:

$$C_1 = C_2 = 0.20264\mu F; C_o = 0.062374\mu F; L_o = 183.601\mu H; R = 5.768\Omega$$

The push-pull class-E amplifier operates in optimum operating condition with the above designed circuit element values.

3.6 Exact Analysis of Push-Pull Class-E Amplifier With Three Modes of Operation (Using State-Equations)

The push-pull class-E amplifier shown in Fig. 3.1 is to be analyzed with three modes of operation (shown in Fig. 3.4). In this case also, the symmetry of the circuit has been maintained. There are six variables $I_1, I_2, I_o, V_{c1}, V_{c2}$ and V_{co} associated with six energy storage circuit elements L_1, L_2, L_o, C_1, C_2 and C_o respectively. Circuit equations have been written for three modes of operation.

Circuit Equations for Mode-1

Equivalent circuit diagram for mode-1 is shown in Fig. 3.5. For this mode switch S1 is "off" and S2 is "on". During "on" period the switch S2 conducts forward current and diode D2 conducts reverse current. The voltage V_{c2} is zero for this mode. There are five first order differential equations for this state.

$$\frac{dI_1}{dt} = \frac{V_s - V_{c1}}{L_1} \tag{3.51}$$

$$\frac{dV_{c1}}{dt} = \frac{I_1 - I_o}{C_1} \tag{3.52}$$

$$\frac{dI_2}{dt} = \frac{V_s}{L_2} \tag{3.53}$$

$$\frac{dI_o}{dt} = \frac{V_{c1} - V_{co}}{L_o} - \frac{R}{L_o} I_o \tag{3.54}$$

$$\frac{dV_{co}}{dt} = \frac{I_o}{C_o} \tag{3.55}$$

Circuit Equations for Mode-2

For mode-2 the switches S1 and S2 are in "on" state as shown in Fig. 3.14. Due to presence of antiparallel diodes, the switches will be conducting in bidirection. For this case both V_{c1} *and* V_{c2} are zero. There are four first order differential equations for this mode.

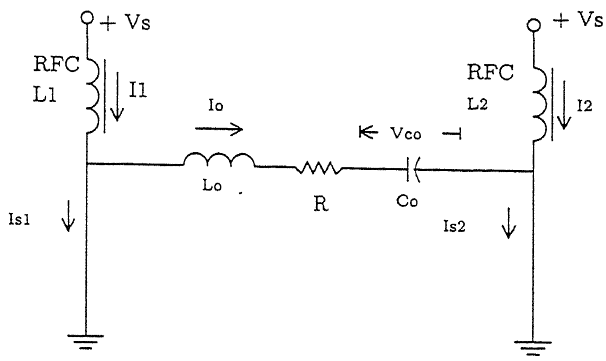

Figure 3.14: Equivalent circuit for mode-2 in three mode operation.

$$\frac{dI_1}{dt} = \frac{V_s}{L_1} \tag{3.56}$$

$$\frac{dI_2}{dt} = \frac{V_s}{L_2} \tag{3.57}$$

$$\frac{dI_o}{dt} = -\frac{V_{co}}{L_o} - \frac{R}{L_o} I_o \tag{3.58}$$

$$\frac{dV_{co}}{dt} = \frac{I_o}{C_o} \tag{3.59}$$

Circuit Equations for Mode-3

For mode-3 the switch state is S1 "on" and S2 "off", it is same as mode-2 of "two mode operation" as shown in Fig. 3.6. The five first order differential equations have been written. For this state V_{c1} is zero.

$$\frac{dI_1}{dt} = \frac{V_s}{L_1} \tag{3.60}$$

$$\frac{dI_2}{dt} = \frac{V_s - V_{c2}}{L_2} \tag{3.61}$$

$$\frac{dV_{c2}}{dt} = \frac{I_2 + I_o}{C_2} \tag{3.62}$$

$$\frac{dI_o}{dt} = -\frac{V_{c2} + V_{co}}{L_o} - \frac{R}{L_o} I_o \tag{3.63}$$

$$\frac{dV_{co}}{dt} = \frac{I_o}{C_o} \tag{3.64}$$

Following optimum circuit design values have been taken.

$$L1 = L2 = 5.0mH; C1 = C2 = 0.03168\mu F; Co = 0.02839\mu F$$
$$Lo = 381.0682\mu H; R = 11.9716\Omega; Vs = 100.0V; f = 50KHz; D = \frac{2}{3}$$

For three modes of operation the duty cycle will be greater than 50-percent. For present case the duty-cycle is chosen as $\frac{2}{3}$. The circuit operates for $\frac{1}{3}$ of time-period in mode-1, then $\frac{1}{6}$ of time-period in mode-2, then $\frac{1}{3}$ of time-period in mode-3 and again in mode-2 for $\frac{1}{6}$ of time-period, then again in mode-1 for $\frac{1}{3}$ of time-period, like this the cycle repeats. The simultaneous differential equations for the three-modes have-been solved with above circuit values, and the steady-state values of circuit parameters $(I_1, I_2, I_o, V_{c1}, V_{c2}, V_{co})$ are shown in Fig. 3.15 to 3.18. The flow-chart and computer program for solution of the exact analysis of three modes of operation are given in Appendix-(E).

Fig. 3.15 is plot of V_{c1} *and* V_{c2} for one time-period at steady-state. The peak values of V_{c1} *and* V_{c2} is 549.12V. Switch turn-on is achieved at zero magnitude and zero slope of V_{c1} *and* V_{c2}. Fig. 3.16 is curve of voltage

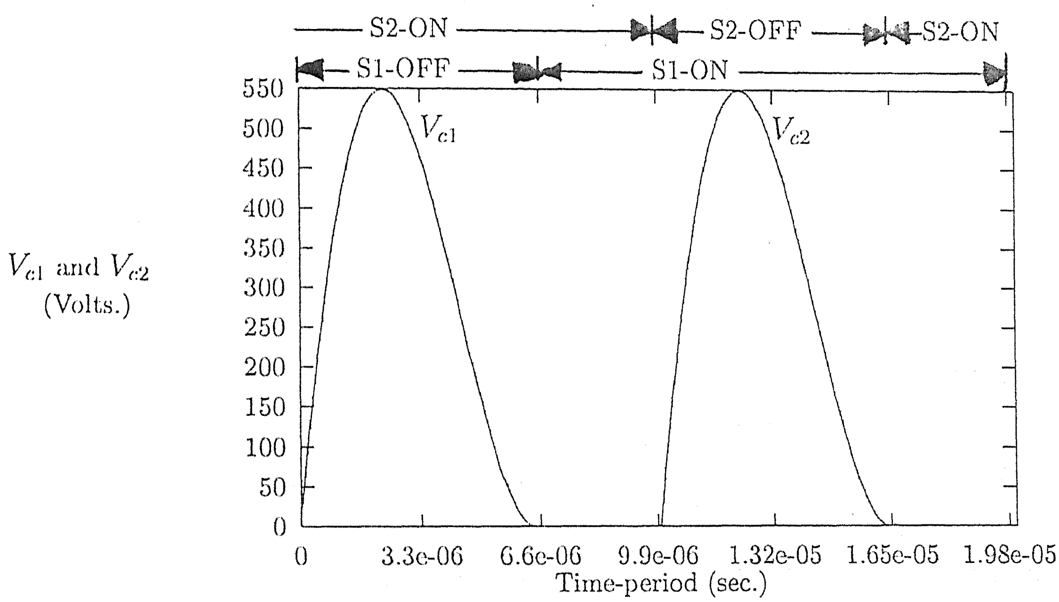

Figure 3.15: Plots of V_{c1} *and* V_{c2} in three mode operation.

V_{co} for one time-period at steady-state. This is the voltage across series capacitor C_o. It is almost sinusoidal with peak magnitude of 2862.2V.

Fig. 3.17 is plots of currents I_1, I_2, I_{s1}, *and* I_{s2} for one time-period at steady-state. Curents I_1 *and* I_2 are constant at 20.2A. Current I_{s1} starts with 15.87A, this is its $I_{s1_{off}}$ value. At switch turn-on time, value of I_{s1} is zero. The peak value I_{s1peak} is 46.34A. The current I_{s2} is same as I_{s1} in magnitude but opposite in phase.

Fig. 3.18 is plot of output current I_o for one cycle at steady-state. It is almost sinusoidal with its peak magnitude of 26.22A. It starts with a value 4.36A with positive phase angle.

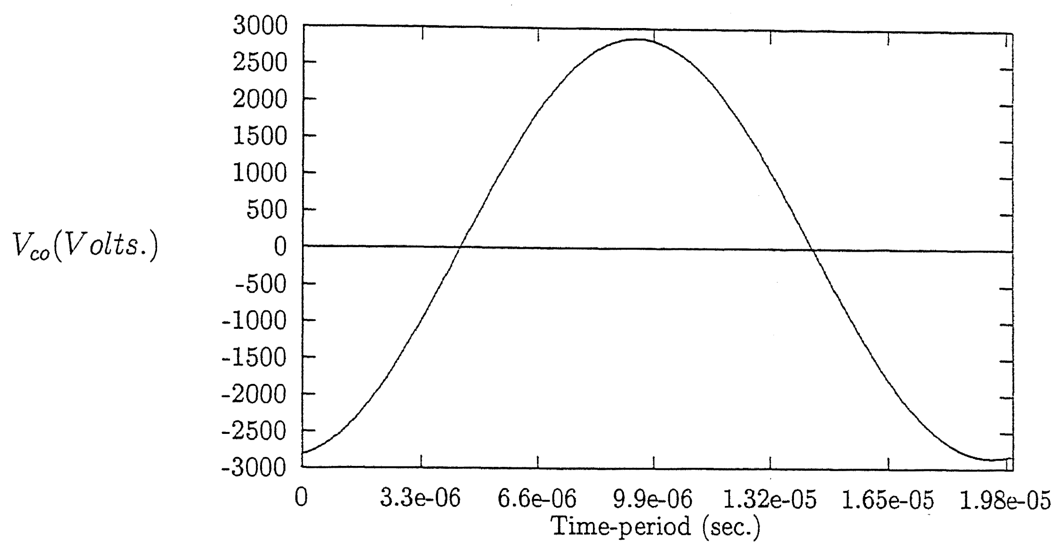

Figure 3.16: Plot of V_{co} in three mode operation.

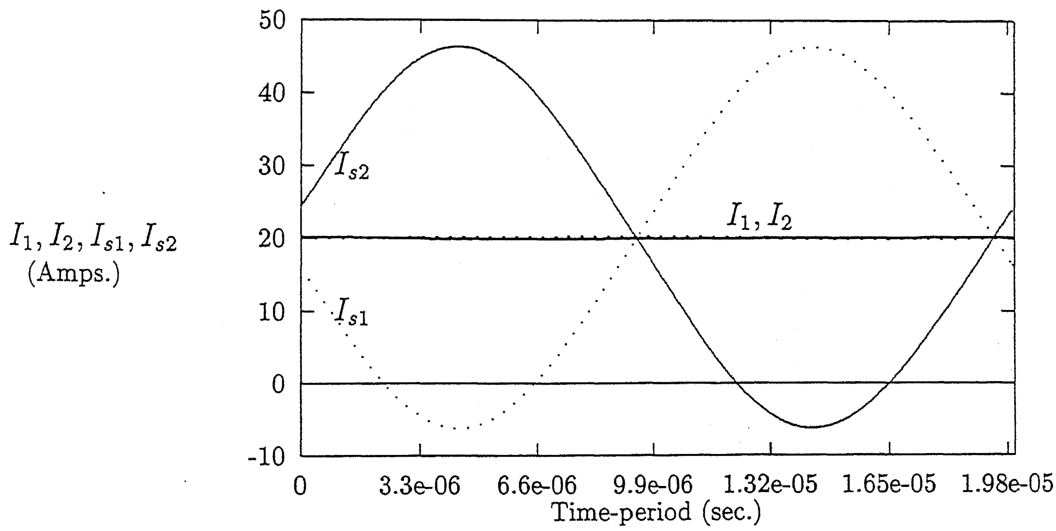

Figure 3.17: Plots of $I_1, I_2, Is1$ *and* I_{s2} in three mode operation.

3.7 Harmonic Analysis of Output Current For Three-Modes of Operation

The push-pull class-E amplifier with three modes of operation retains symmetry in its output current waveform. Hence, because of symmetry, the even harmonics will be absent. Using "Discrete Fourier Transform Method"

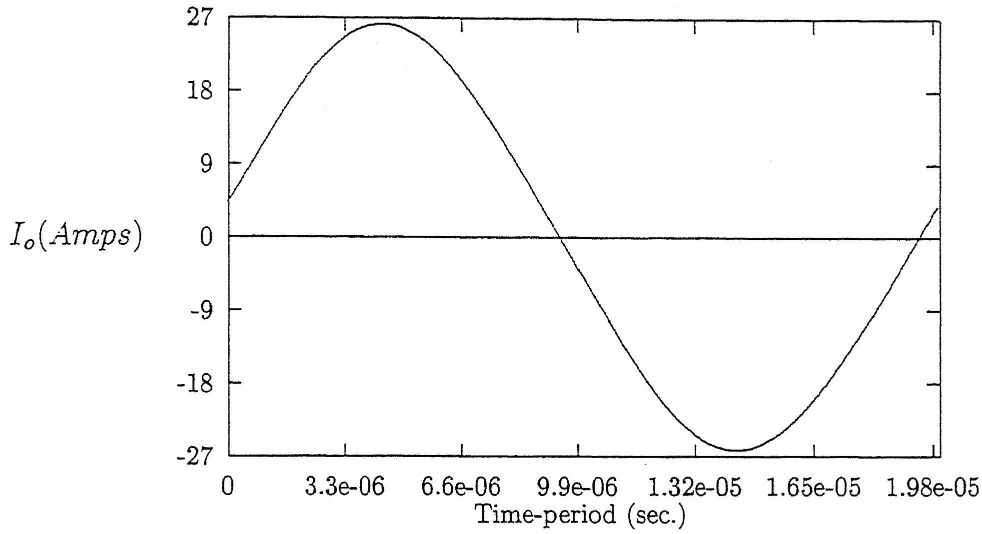

Figure 3.18: Plot of I_o in three mode operation.

(NAG subroutine C06EAF), the harmonic components of the output current waveform I_o (shown in Fig.3.18) are obtained. The magnitudes and phase angles are tabulated in Table 3.2.

Table-3.2		
Harmonics and Phases of Output Current		
Harmonic	Peak magnitude (Amp.)	Phase angle (Degree)
1	25.839	10.77
3	0.5497	226.26
5	0.05708	194.50
7	0.02819	189.89
9	0.01059	199.80
11	0.00506	184.32
13	0.00341	189.48

From Table-3.2, it is evident that third and higher harmonics are very small in magnitude in comparison to fundamental harmonic component. This indicates that the output current I_o is almost sinusoidal.

3.8 Analysis Assuming Constant Input Current and Sinusoidal Output Current With Three Modes of Operation

3.8.1 Analysis

For analysis of push-pull class-E amplifier (shown in Fig. 3.1) with three modes of operation. all assumptions are same as mentioned in section 3.4.1. Since, the operating frequency is different from the resonant frequency of L_o and C_o, the equivalent circuit shown in Fig. 3.12 is valid for this analysis. The output current I_o is assumed sinusoidal. Consider,

$$I_o(\theta) = I_{om}\sin(\theta + \phi) \tag{3.65}$$

Output voltage is,

$$V_R(\theta) = I_{om}R\sin(\theta + \phi) \tag{3.66}$$

Here, θ(=wt) is "angular time" and ϕ is phase angle. For three modes of circuit operation, the equivalent circuit shown in Fig. 3.13 is assumed. The series tuned circuit consisting of R, L_o and C_o is excited by a voltage source value $V_c(\theta)$. Where, $V_c(\theta)$ for different modes (Fig. 3.4) is defined as following:

$$\begin{aligned} V_c(\theta) &= V_{c1}(\theta) \text{ from 0 to } \theta_1 \text{ for mode-1} \\ &= 0\ from\ \theta_1\ to\ \pi \text{ for mode-2} \\ &= \text{-}V_{c2}(\theta)\ from\ \pi\ to\ (\pi + \theta_1) \text{ for mode-3} \\ &= 0 \text{ from } (\pi + \theta_1) \text{ to } 2\pi \text{ for mode-2} \end{aligned}$$

Hence,

$$V_c(\theta) = V_{c1}(\theta) - V_{c2}(\theta) \tag{3.67}$$

$V_c(\theta)$ can also be expressed by;

$$V_c(\theta) = I_{om}\sqrt{(R^2 + X^2)}\sin(\theta + \phi + \psi) \tag{3.68}$$

Where, $\psi = \arctan\left(\frac{X}{R}\right)$.

For the duration 0 to θ_1 S1 "off" and S2 "on"(mode-1) $V_{c2}(\theta)$ is zero, and

$$V_{c1}(\theta) = \frac{1}{wC_1}\int_0^{\theta}[I_1 - I_{om}\sin(u+\phi)]du \tag{3.69}$$

Solving above,

$$V_{c1}(\theta) = \frac{1}{wC_1}[I_1\theta + I_{om}\cos(\theta+\phi) - I_{om}\cos\phi] \tag{3.70}$$

Where, θ is between 0 and θ_1.

For the duration π to $(\pi+\theta_1)$ S1 "on" and S2 "off"(mode-3) the $V_{c1}(\theta)$ is zero, and

$$V_{c2}(\theta) = \frac{1}{wC_2}\int_{\pi}^{\theta}[I_2 + I_{om}\sin(u+\phi)]du \tag{3.71}$$

Solving above,

$$V_{c2}(\theta) = \frac{1}{wC_2}[I_2(\theta-\pi) - I_{om}\cos(\theta+\phi) - I_{om}\cos\phi] \tag{3.72}$$

Where, θ is from π to $(\pi+\theta_1)$.

The average voltage across shunt capacitors C_1 or C_2 is equal to source voltage V_s, because average voltage across L_1 or L_2 is zero. Therefore, source voltage,

$$V_s = \frac{1}{2\pi}\int_0^{\theta_1} V_{c1}(\theta)d\theta \tag{3.73}$$

From equations (3.70) and (3.73), we get,

$$V_s = \frac{1}{2\pi wC_1}[\frac{\theta_1^2}{2}I_1 + I_{om}\sin(\theta_1+\phi) - I_{om}\sin\phi - \theta_1 I_{om}\cos\phi] \tag{3.74}$$

Similarly, the average voltage across C_2 is given by,

$$V_s = \frac{1}{2\pi wC_2}[\frac{\theta_1^2}{2}I_2 + I_{om}\sin(\theta_1+\phi) - I_{om}\sin\phi - \theta_1 I_{om}\cos\phi] \tag{3.75}$$

For symmetry, we assume, in the following, equal voltage sources V_s: equal shunt capacitors C_1 and C_2, equal RF Choke inductors L_1 and L_2 at the two ends of the circuit. The voltage across the series impedance Z_o: is $V_c(\theta)$. Where, $V_c(\theta)$ is given by Eq. (3.67). Fundamental component of $V_c(\theta)$ is obtained as,

$$V_{c_{fundamental}}(\theta) = a_1 \cos\theta + b_1 \sin\theta \tag{3.76}$$

Where,

$$a_1 = \frac{1}{\pi}\left[\int_0^{\theta_1} V_{c1}(\theta)\cos\theta d\theta - \int_{\pi}^{(\pi+\theta_1)} V_{c2}(\theta)\cos\theta d\theta\right] \tag{3.77}$$

and,

$$b_1 = \frac{1}{\pi}\left[\int_0^{\theta_1} V_{c1}(\theta)\sin\theta d\theta - \int_{\pi}^{(\pi+\theta_1)} V_{c2}(\theta)\sin\theta d\theta\right] \tag{3.78}$$

Where, $V_{c1}(\theta)$ and $V_{c2}(\theta)$ are substituted from Eq. (3.70) and (3.72), respectively. Solving Eq. (3.77) and (3.78), we get,

$$\begin{aligned} a_1 &= \frac{1}{\pi w C_1}[2I_1(\theta_1 \sin\theta_1 + \cos\theta_1 - 1) + (\theta_1 + \frac{1}{2}\sin 2\theta_1 - 2\sin\theta_1)I_{om}\cos\phi \\ &\quad + \frac{1}{2}(\cos 2\theta_1 - 1)I_{om}\sin\phi] \end{aligned} \tag{3.79}$$

and,

$$\begin{aligned} b_1 &= \frac{1}{\pi w C_1}[2I_1(-\theta_1 \cos\theta_1 + \sin\theta_1) + (2\cos\theta_1 - \frac{1}{2}\cos 2\theta_1 - \frac{3}{2})I_{om}\cos\phi \\ &\quad + (\frac{1}{2}\sin 2\theta_1 - \theta_1)I_{om}\sin\phi] \end{aligned} \tag{3.80}$$

Simplifying the Eq. (3.68), we get,

$$V_c(\theta) = (RI_{om}\sin\phi + XI_{om}\cos\phi)\cos\theta + (RI_{om}\cos\phi - XI_{om}\sin\phi)\sin\theta \tag{3.81}$$

Equating coefficients of $\cos\theta$ and $\sin\theta$, from Eq. (3.76) and (3.80), and substituting values a_1 and b_1 from Eq. (3.79) and (3.80), we obtain following equations:

$$2I_1(\theta_1 \sin\theta_1 + \cos\theta_1 - 1) + (\theta_1 + \tfrac{1}{2}\sin 2\theta_1 - 2\sin\theta_1 - \pi w C_1 X) I_{om}\cos\phi$$
$$+\{\tfrac{1}{2}(\cos 2\theta_1 - 1) - \pi w C1R\} I_{om}\sin\phi = 0 \qquad (3.82)$$

and,

$$2I_1(-\theta_1\cos\theta_1 + \sin\theta_1) + (2\cos\theta_1 - \tfrac{1}{2}\cos 2\theta_1 - \tfrac{3}{2} - \pi w C_1 R) I_{om}\cos\phi$$
$$+(\pi w C_1 X + \tfrac{1}{2}\sin 2\theta_1 - \theta_1) I_{om}\sin\phi = 0 \qquad (3.83)$$

Using Eq. (3.74), (3.82) and (3.83), we form the following matrix equation:

$$\begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_{om}\sin\phi \\ I_{om}\cos\phi \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2\pi w C_1 V_s \end{bmatrix} \qquad (3.84)$$

Where,

$$X_{11} = 2(\theta_1\sin\theta_1 + \cos\theta_1 - 1); X_{12} = \tfrac{1}{2}(\cos 2\theta_1 - 1) - \pi w C1R$$
$$X_{13} = \theta_1 + \tfrac{1}{2}\sin 2\theta_1 - 2\sin\theta_1 - \pi w C_1 X$$
$$X_{21} = 2(-\theta_1\cos\theta_1 + \sin\theta_1); X_{22} = \pi w C_1 X + \tfrac{1}{2}\sin 2\theta_1 - \theta_1$$
$$X_{23} = 2\cos\theta_1 - \tfrac{1}{2}\cos 2\theta_1 - \tfrac{3}{2} - \pi w C_1 R$$
$$X_{31} = \tfrac{\theta_1^2}{2}; X_{32} = \cos\theta_1 - 1; X_{33} = \sin\theta_1 - \theta_1$$

From above matrix Eq. (3.84), we can solve for I_1, I_{om} and ϕ, for a given set of circuit element values. The angle ϕ is obtained as (with $\theta_1 = \frac{2\pi}{3}$),

$$\phi = \arctan\frac{1.8191374 - 19.154173 w C_1 X + \pi w C_1 R}{2.0453143 + 19.154173 w C_1 R + \pi w C_1 X} \qquad (3.85)$$

Analysis For Optimum Operation

For optimum operation of the circuit the magnitudes and slopes of $V_{c1}(\theta)$ and $V_{c2}(\theta)$ are zero at switch turn-on, i.e., at $\theta = \theta_1$ and $\theta = (\pi + \theta_1)$, respectively. For $D = \frac{2}{3}$, the angular angle $\theta_1 = \frac{2\pi}{3}$. Therefore,

$$V_{c1}(\theta)/_{\theta=\frac{2\pi}{3}} = 0 = I_1\theta + I_{om}\cos(\theta+\phi) - I_{om}\cos\phi \qquad (3.86)$$

Which gives,

$$\frac{2\pi}{3}I_1 - 1.5I_{om}\cos\phi - 0.8660254I_{om}\sin\phi = 0 \tag{3.87}$$

The slope of $V_{c1}(\theta)$ at the time of turn "on" of S1 must also be zero for optimum operation. Hence,

$$\frac{dV_{c1}(\theta)}{dt}/_{\theta=\frac{2\pi}{3}} = 0 = I_1 - I_{om}\sin(\theta+\phi) \tag{3.88}$$

Above equation can be written as,

$$I_1 - 0.8660254I_{om}\cos\phi + 0.5I_{om}\sin\phi = 0 \tag{3.89}$$

Eq. (3.87) and (3.89) give following equations:

$$\cos\phi = 1.275482\frac{I_1}{I_{om}} \tag{3.90}$$

and,

$$\sin\phi = 0.2091995\frac{I_1}{I_{om}} \tag{3.91}$$

The above two equations yield,

$$\phi = 9.3144939^\circ = 0.1625685^c \tag{3.92}$$

Because of symmetry, we get the same phase angle ϕ by applying zero magnitude and zero slope condition on $V_{c2}(\theta)$.

From Eqs. (3.74), (3.90) and (3.91), we get

$$\frac{V_s}{I_1} = \frac{0.3126826}{2\pi w C_1} \tag{3.93}$$

Similarly, because of symmetry of the circuit, following can also be obtained.

$$\frac{V_s}{I_2} = \frac{0.3126826}{2\pi w C_2} \tag{3.94}$$

From Eq. (3.91) and (3.92),

$$I_{om} = 1.292524 I_1 \tag{3.95}$$

Peak switch current is given by,

$$I_{s1pk} = I_1 + I_{om} \tag{3.96}$$

From Eq. (3.95) and (3.96), we get

$$I_{s1pk} = 2.292524 I_1 \tag{3.97}$$

For the value of θ_{vmax} at which the value of $V_{c1}(\theta)$ is maximum, from Eq. (3.70);

$$\frac{dV_{c1}(\theta)}{dt}/_{\theta=\theta_{vmax}} = 0 = I_1 - I_{om}\sin(\theta_{vmax} + \phi) \tag{3.98}$$

Using Eq. (3.91), (3.92) and (3.98), we get

$$\theta_{vmax} = 41.371^{\circ} = 0.72206^{c} \tag{3.99}$$

Substituting value of θ_{vmax} in place of θ in Eq. (3.70) and using Eq. (3.90), (3.91) and (3.93), we obtain,

$$V_{c1_{max}} = 5.335 V_s \tag{3.100}$$

Similarly, we can also obtain,

$$V_{c2_{max}} = 5.335 V_s \tag{3.101}$$

Output power is given by

$$P_o = \frac{I_{om}^2 R}{2} \tag{3.102}$$

Input power is,

$$P_i = V_s(I_1 + I_2) \tag{3.103}$$

Efficiency of the amplifier is,

$$\eta = \frac{I_{om}^2 R}{2V_s(I_1 + I_2)} \tag{3.104}$$

3.8.2 Comparison of Approximate Analysis Results with Results of Simulations

The results obtained by above analysis will be approximate because the output current is assumed sinusoidal and input current is assumed constant. For a given value of $V_s = 100V, f = 50KHz$ and $C_1 = C_2 = 0.03168\mu F$, we calculate I_1 and I_2 20A from Eq. (3.93) and (3.94). The simulation result in Fig. 3.17 shows its value 20.2A.

The peak output current I_{om} given by Eq. (3.95) is calculated as 25.85A. The simulation result in Fig. 3.18 shows 26.22A.

Peak switch current I_{s1pk} is calculated as 45.85A from Eq. (3.97) and simulation result gives 46.336A, as in Fig. 3.17.

The values of V_{c1max} and V_{c2max} are calculated as 533.5V from Eq. (3.100) and (3.101). The simulation result gives $V_{c1max} = V_{c2max} = 549.12V$ from Fig. 3.15.

3.9 Design of Push-Pull Class-E Amplifier For Optimum Operation at $D = \frac{2}{3}$

3.9.1 Design Equations

For symmetrical operation of circuit, we have to keep $C_1 = C_2$ and $L_1 = L_2$. The supply voltage is common to both ends. Net resistive load offered by the amplifier at each supply end is,

$$R_{dc} = \frac{2V_s^2}{P_o} \tag{3.105}$$

Where, P_o is desired output power at unity efficiency.

Also,

$$R_{dc} = \frac{V_s}{I_1} \tag{3.106}$$

From Eq. (3.104) with $I_1 = I_2$ and $\eta = 1$, we get,

$$V_s = \frac{I_{om}^2 R}{4I_1} \tag{3.107}$$

Hence, substituting V_s from Eq. (3.106) and using Eq. (3.91) and (3.92), we have,

$$R = 2.3943226 R_{dc} \tag{3.108}$$

Using Eq. (3.93) and (3.106), we have,

$$C_1 = \frac{0.3126826}{2\pi w R_{dc}} \tag{3.109}$$

For a given quality factor Q_l,

$$L_o = \frac{Q_l R}{w} \tag{3.110}$$

Substituting calculated values of R, C_1, f, L_o (from above equations),$\theta_1 = \frac{2\pi}{3}$ and ϕ (from Eq. (3.92)) in Eq. (3.85), we can obtain following equation for C_o;

$$C_o = \frac{1}{w^2 L_o}[1 + \frac{0.6330537}{Q_l - 0.6330537}] \tag{3.111}$$

Using Eq. (3.105)—(3.111) the circuit elements can be designed for optimum operating condition.

3.9.2 Design Example

The design problem of Sec. 3.5.2 has been repeated here. The amplifier is to deliver 4KW at 50 KHz,$D = \frac{2}{3}$, $V_s = 100V$, $L_1 = L_2 = 5.0mH$ (RF Choke), and $Q_l = 10$.

Using design equations of previous section the designed values obtained are given below:

$$C_1 = C_2 = 0.03168\mu F; C_o = 0.02839\mu F$$
$$L_o = 381.0682\mu H; R = 11.9716\Omega$$

The class-E push-pull amplifier operates in optimum operating condition with the above designed circuit element values for three modes of operation. This design has been simulated and the results have been shown in Sec. 2.6.

Chapter 4

Conclusion

Based on design and analysis carried out, following conclusions can be drawn:

- Simulation study of basic class-E amplifier shows that, during transient period the shunt capacitor voltage V_{c1} and out put current I_o increases exponentially and reaches the steady-state in around 400 cycles. At steady-state, peak voltage across the switch is 3.587 times the supply voltage. Peak switch current is 2.8 times the input current from the source. For optimum operation, at the time of switch turn-on both the voltage across switch and the slope of the voltage is zero. At the time of switch turn-off the switch current is around 2.16 times the source input current.

- Power output of the class-E amplifier follows the square law with voltage source, this is because, an equivalent resistance R_{dc} can replace the amplifier across the supply voltage at optimum operating condition.

- If the load resistance is decreased from its optimum value, the power output to the load is decreased, and the circuit operates in underdamped suboptimum operating condition. Increase in the resistance makes the circuit operation in overdamped suboptimum operating and power output decreases. The power output is maximum only in optimum operating condition.

- The basic class-E amplifier, when it is used for high frequency induction heating, generally the load resistance increases and load inductance decreases with temperature. For a given characteristic of change of R and L_o with temperature, the circuit can be designed to operate in underdamped suboptimum operating condition. Underdamped suboptimum operating condition also has 100% efficiency.

- For underdamped suboptimum operating condition, the voltage across the switch will be much larger than that in optimum operating condition.
- Since class-E amplifier consists of tuned $R - L_o - C_o$ circuit, a small change in operating frequency causes large change in output power.
- Harmonic analysis of the basic class-E amplifier load current and voltage across the switch, shows that all harmonics are present. At optimum operating condition the second and third harmonics of output current are 5.8% and 0.86% of the fundamental. Only second harmonic is significant.
- An analysis assuming constant input current and sinusoidal output current has been carried out for solutions of basic class-E amplifier for a given set of circuit elements and for any duty-cycle.
- Analysis assuming constant input current and sinusoidal output current for optimum operating condition at 50% duty-cycle has been done. The analysis assuming constant input current and sinusoidal output current is an approximate analysis, but this gives design equations to obtain approximate values of circuit elements.
- Analysis assuming constant input current and non-sinusoidal output current has also been carried out. This analysis includes finite number of harmonic components in the output current, leading to more accurate analysis. For a given set of circuit elements the accurate solutions can be obtained for any duty-cycle.
- A comparison of the analysis assuming sinusoidal output current and the analysis assuming non-sinusoidal output current with the exact analysis has been done. The analysis assuming non-sinusoidal output current gives results very close to those obtained by the exact analysis, using state equations.
- For a given operating frequency, output power, supply voltage and quality-factor the approximate circuit element values can be obtained for D=0.5. A comparative study of different quality-factor on circuit responses shows that, if we choose higher quality-factor, the approximate designed circuit values become more accurate. This is because, with higher quality-factor the output current becomes more sinusoidal.
- A modified procedure for more accurate design for any quality-factor has been contributed. This has two design options:
    1. Design for a given P_o, Q_l and f
    2. Design for a given R, Q_l and f

- "Modified procedure for more accurate design" assumes finite number of harmonic components in the output current. Considering different number of harmonics (in output current), the circuit design values "With More Accurate Procedure" were obtained. A comparative study of the circuit responses with the different circuit values have been done. It shows that, assuming first and second harmonics in output current is sufficient to obtain accurate optimum circuit element values, but as more number of harmonics were considered in the output current, more accurate design values were obtained.

- Class-E push-pull amplifier have analysis and design procedure similar to that of the basic class-E amplifier. The push-pull class-E amplifier operating in optimum condition with 50% duty-cycle has peak voltage across the switch 3.58 times the supply voltage. Peak switch current is 2.86 times the input source current. The current through the switch at the instant of switch-off is twice the input source current.

- Harmonic analysis shows that all even harmonic components are absent in the output current. Third and higher order of harmonic components are negligible in comparison to fundamental. Third harmonic component is 0.78% of the fundamental.

- Analysis assuming constant input current and sinusoidal output current with D=0.5 has been carried out. Results of this analysis and results of the exact analysis using state equations were compared, and both results have almost the same value. The analysis assuming constant input current and sinusoidal output current for optimum operating condition gave equations for the design of circuit element values.

- An analysis and design of push-pull class-E amplifier was carried out with $D = \frac{2}{3}$. This has three modes of operation. At optimum operating condition peak voltage across the switch is 5.49 times the supply voltage and peak switch current is 2.29 times the input source current. Power capability of the switch have been reduced, but the switch current at the instant of turn-off is only 0.79 times the input source current.

- Harmonic analysis of load current shows that even harmonics are absent and third harmonic is only 2.1% of the fundamental.

- Analysis assuming constant input current and sinusoidal output current with $D > 0.5$ have been done. This analysis for optimum operating condition (with $D = \frac{2}{3}$) leads an accurate design of the circuit.

Suggestions For The Future Work

1. A study can be carried out for implementing the class-E amplifier as DC to DC converter by changing the duty-cycle (at an operating frequency) such that the circuit operation remains either in optimum or in underdamped suboptimum.

2. A study can be done to bring the class-E amplifier operation back to optimum by changing either duty-cycle or operating frequency, if due to small change in circuit element values, the optimum operation is disturbed.

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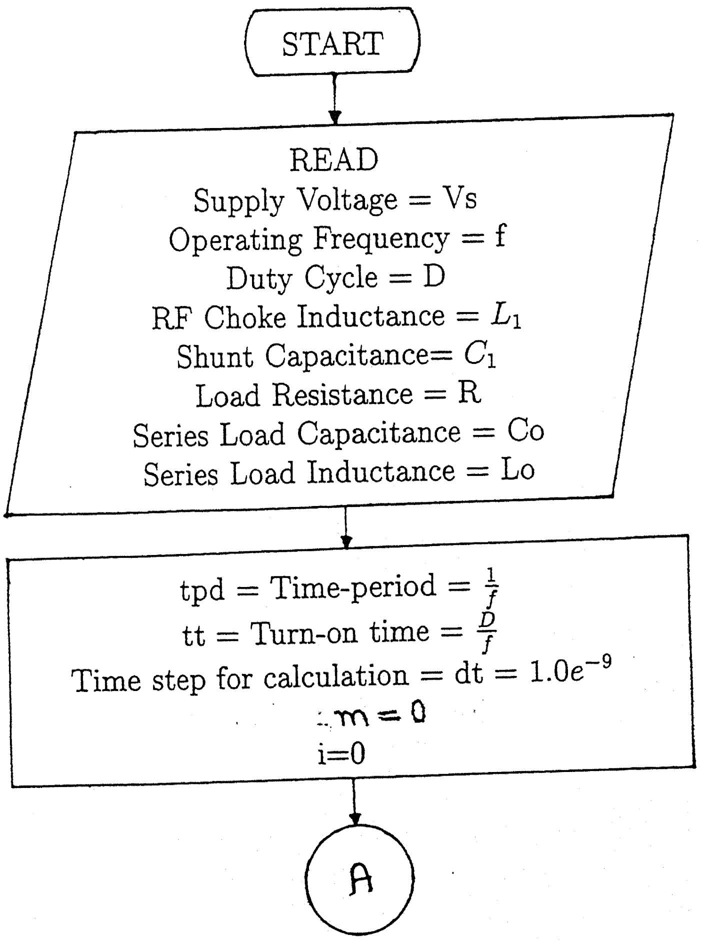
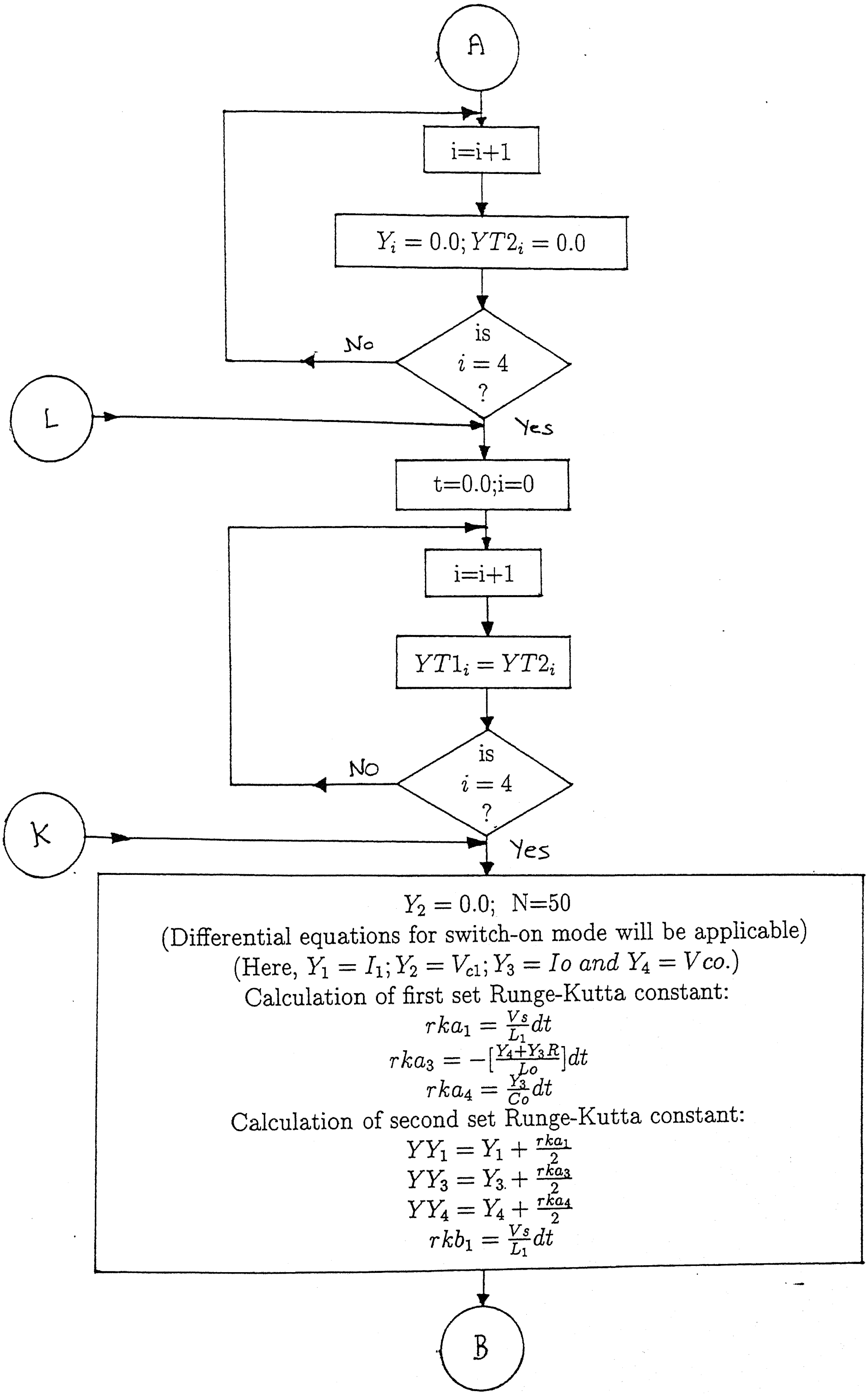
Appendix A

Flow-Chart For Exact Analysis of Class-E Amplifier (With Diode), Using State Equations




A
i=i+1
$Y_i = 0.0; YT2_i = 0.0$
is $i = 4$?
No
Yes
L
t=0.0;i=0
i=i+1
$YT1_i = YT2_i$
is $i = 4$?
NO
Yes
K
$Y_2 = 0.0$; N=50
(Differential equations for switch-on mode will be applicable)
(Here, $Y_1 = I_1; Y_2 = V_{c1}; Y_3 = Io$ and $Y_4 = Vco$.)
Calculation of first set Runge-Kutta constant:
$rka_1 = \frac{Vs}{L_1}dt$
$rka_3 = -[\frac{Y_4+Y_3R}{Lo}]dt$
$rka_4 = \frac{Y_3}{Co}dt$
Calculation of second set Runge-Kutta constant:
$YY_1 = Y_1 + \frac{rka_1}{2}$
$YY_3 = Y_3 + \frac{rka_3}{2}$
$YY_4 = Y_4 + \frac{rka_4}{2}$
$rkb_1 = \frac{Vs}{L_1}dt$
B

B

$$rkb_3 = -[\frac{YY_4 + YY_3 R}{Lo}]dt$$
$$rkb_4 = \frac{YY_3}{Co}dt$$

Calculation of third set Runge-Kutta constant:

$$YYY_1 = YY_1 + \frac{rkb_1}{2}$$
$$YYY_3 = YY_3 + \frac{rkb_3}{2}$$
$$YYY_4 = YY_4 + \frac{rkb_4}{2}$$
$$rkc_1 = \frac{Vs}{L_1}dt$$
$$rkc_3 = -[\frac{YYY_4 + YYY_3 R}{Lo}]dt$$
$$rkc_4 = \frac{YYY_3}{Co}dt$$

Calculation of fourth set Runge-Kutta constant:

$$YYYY_1 = YYY_1 + \frac{rkc_1}{2}$$
$$YYYY_3 = YYY_3 + \frac{rkc_3}{2}$$
$$YYYY_4 = YYY_4 + \frac{rkc_4}{2}$$
$$rkd_1 = \frac{Vs}{L_1}dt$$
$$rkd_3 = -[\frac{YYYY_4 + YYYY_3 R}{Lo}]dt$$
$$rkd_4 = \frac{YYYY_3}{Co}dt$$

Solutions at time t=t+dt by Runge-Kutta fourth order method:

$$Y_1 = Y_1 + \frac{1}{6}[rka_1 + 2rkb_1 + 2rkc_1 + rkd_1]$$
$$Y_3 = Y_3 + \frac{1}{6}[rka_3 + 2rkb_3 + 2rkc_3 + rkd_3]$$
$$Y_4 = Y_4 + \frac{1}{6}[rka_4 + 2rkb_4 + 2rkc_4 + rkd_4]$$
$$I_{s1} = Y_1 - Y_3$$

t=t+dt

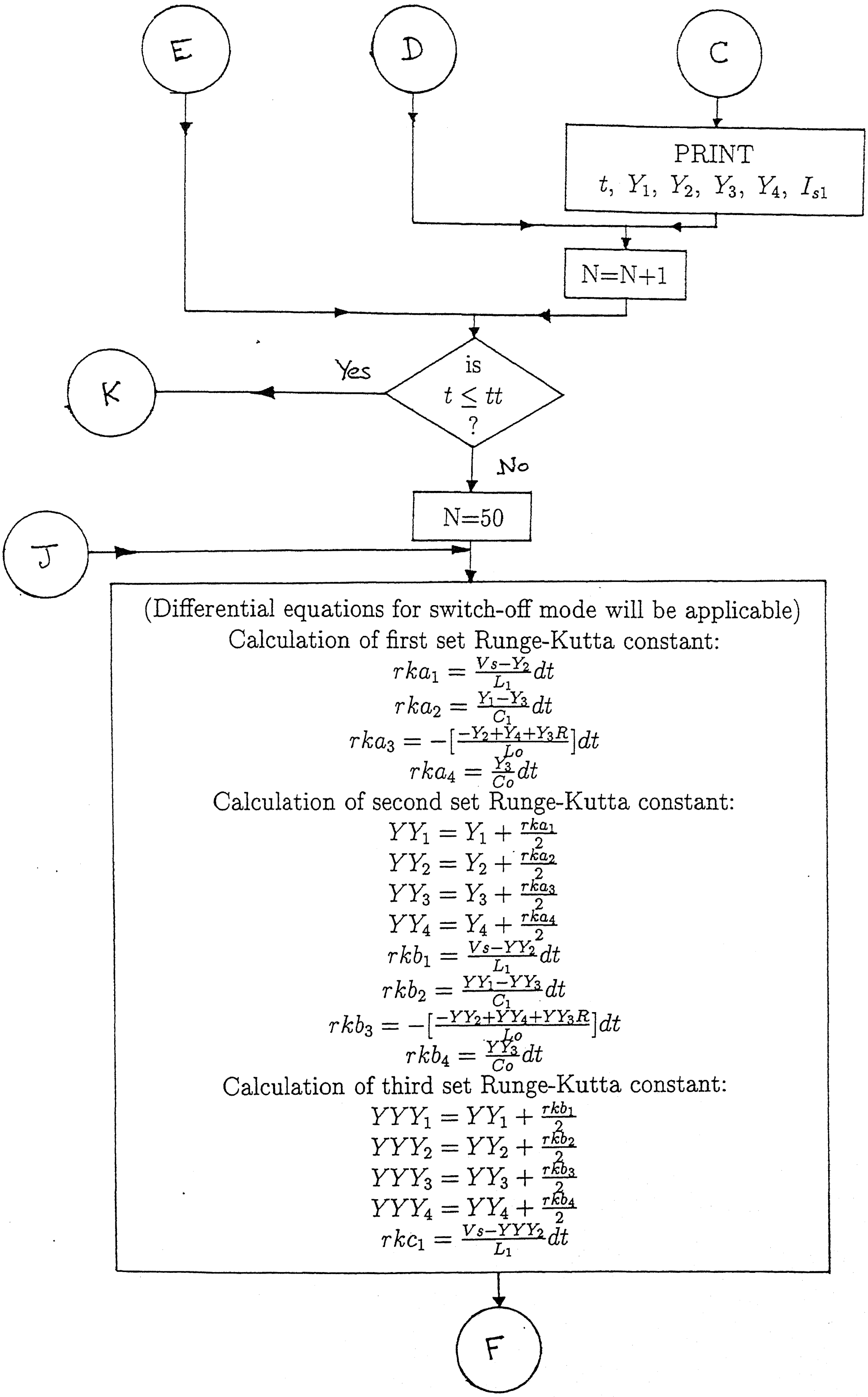
is m=1 ?

No → E

Yes → is N=50 ?

No → D

Yes → N=0 → C


E
D
C
PRINT
$t,\ Y_1,\ Y_2,\ Y_3,\ Y_4,\ I_{s1}$
N=N+1
K
Yes
is $t \leq tt$?
No
N=50
J
(Differential equations for switch-off mode will be applicable)
Calculation of first set Runge-Kutta constant:
$rka_1 = \frac{Vs-Y_2}{L_1}dt$
$rka_2 = \frac{Y_1-Y_3}{C_1}dt$
$rka_3 = -[\frac{-Y_2+Y_4+Y_3R}{Lo}]dt$
$rka_4 = \frac{Y_3}{Co}dt$
Calculation of second set Runge-Kutta constant:
$YY_1 = Y_1 + \frac{rka_1}{2}$
$YY_2 = Y_2 + \frac{rka_2}{2}$
$YY_3 = Y_3 + \frac{rka_3}{2}$
$YY_4 = Y_4 + \frac{rka_4}{2}$
$rkb_1 = \frac{Vs-YY_2}{L_1}dt$
$rkb_2 = \frac{YY_1-YY_3}{C_1}dt$
$rkb_3 = -[\frac{-YY_2+YY_4+YY_3R}{Lo}]dt$
$rkb_4 = \frac{YY_3}{Co}dt$
Calculation of third set Runge-Kutta constant:
$YYY_1 = YY_1 + \frac{rkb_1}{2}$
$YYY_2 = YY_2 + \frac{rkb_2}{2}$
$YYY_3 = YY_3 + \frac{rkb_3}{2}$
$YYY_4 = YY_4 + \frac{rkb_4}{2}$
$rkc_1 = \frac{Vs-YYY_2}{L_1}dt$
F

F

$$rkc_2 = \frac{YYY_1 - YYY_3}{C_1}dt$$
$$rkc_3 = -[\frac{-YYY_2 + YYY_4 + YYY_3 R}{Lo}]dt$$
$$rkc_4 = \frac{YYY_3}{Co}dt$$

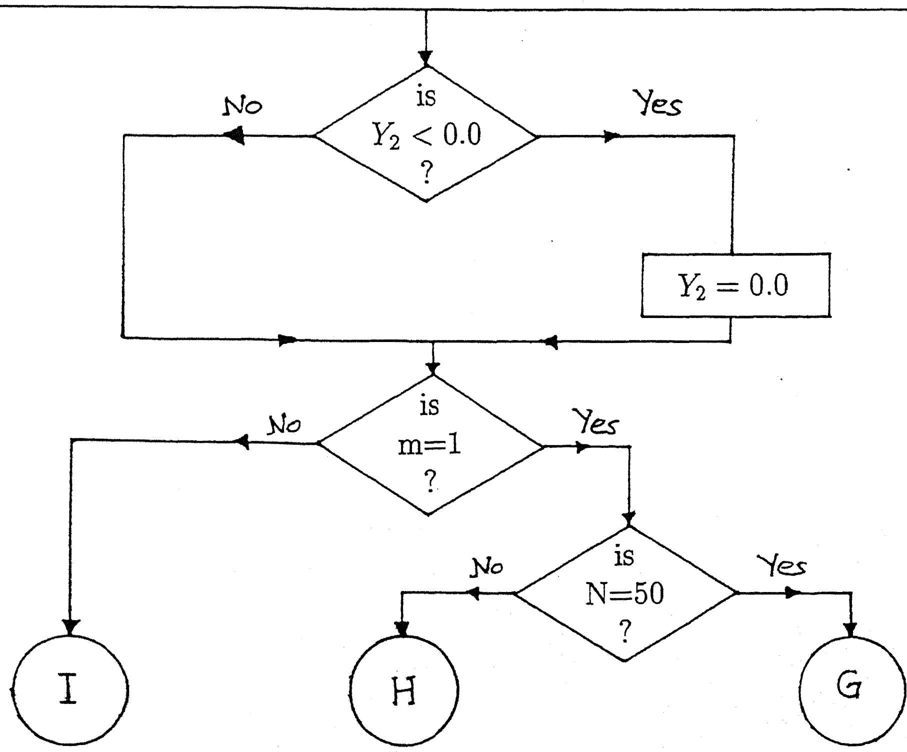
Calculation of fourth set Runge-Kutta constant:

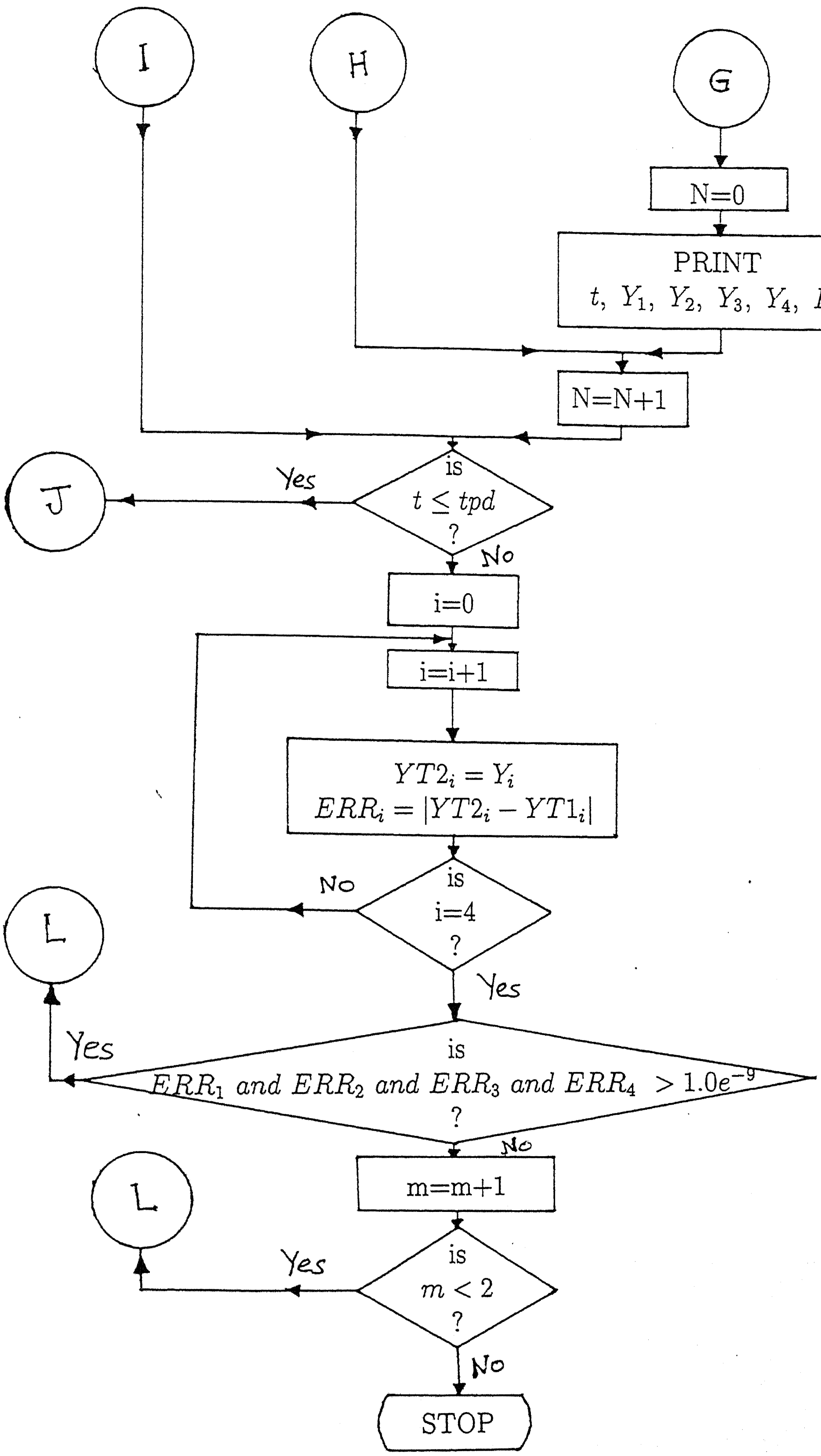
$$YYYY_1 = YYY_1 + \frac{rkc_1}{2}$$
$$YYYY_2 = YYY_2 + \frac{rkc_2}{2}$$
$$YYYY_3 = YYY_3 + \frac{rkc_3}{2}$$
$$YYYY_4 = YYY_4 + \frac{rkc_4}{2}$$
$$rkd_1 = \frac{Vs - YYYY_2}{L_1}dt$$
$$rkd_2 = \frac{YYYY_1 - YYYY_3}{C_1}dt$$
$$rkd_3 = -[\frac{-YYYY_2 + YYYY_4 + YYYY_3 R}{Lo}]dt$$
$$rkd_4 = \frac{YYYY_3}{Co}dt$$

Solutions at time t=t+dt by Runge-Kutta fourth order method:

$$Y_1 = Y_1 + \frac{1}{6}[rka_1 + 2rkb_1 + 2rkc_1 + rkd_1]$$
$$Y_2 = Y_2 + \frac{1}{6}[rka_2 + 2rkb_2 + 2rkc_2 + rkd_2]$$
$$Y_3 = Y_3 + \frac{1}{6}[rka_3 + 2rkb_3 + 2rkc_3 + rkd_3]$$
$$Y_4 = Y_4 + \frac{1}{6}[rka_4 + 2rkb_4 + 2rkc_4 + rkd_4]$$
$$I_{s1} = Y_1 - Y_3$$

t=t+dt



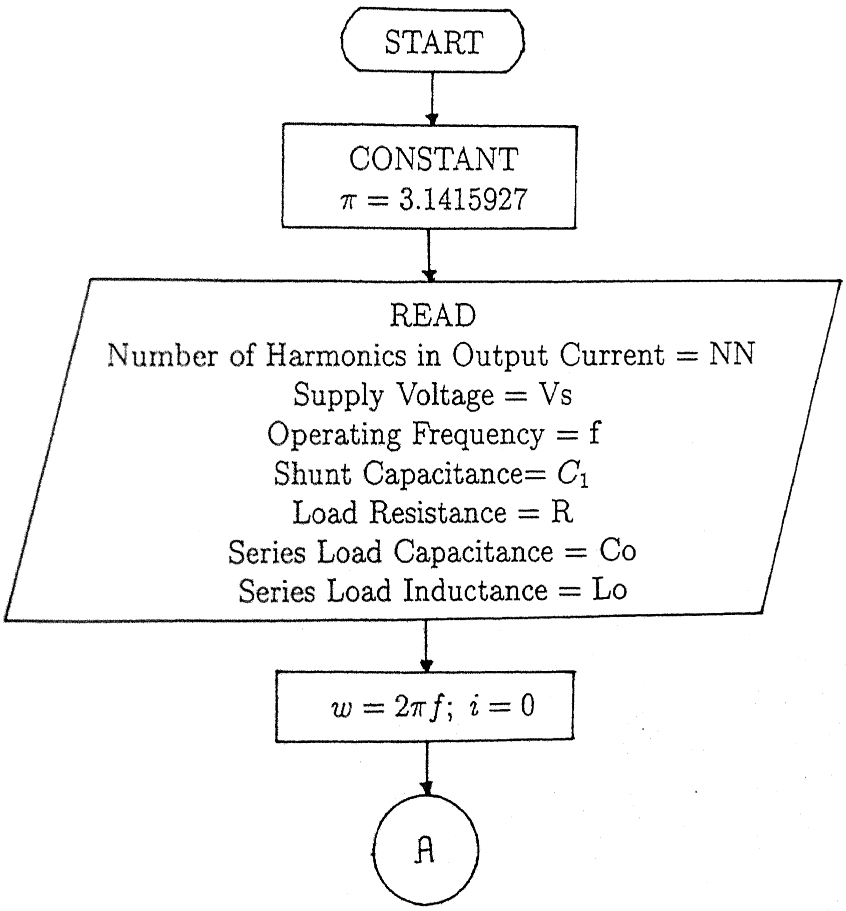

I
H
G
N=0
PRINT
$t,\ Y_1,\ Y_2,\ Y_3,\ Y_4,\ I_{s1}$
N=N+1
J
Yes
is
$t \leq tpd$
?
No
i=0
i=i+1
$YT2_i = Y_i$
$ERR_i = |YT2_i - YT1_i|$
No
is
i=4
?
Yes
L
Yes
is
ERR_1 and ERR_2 and ERR_3 and ERR_4 $> 1.0e^{-9}$
?
No
m=m+1
L
Yes
is
$m < 2$
?
No
STOP

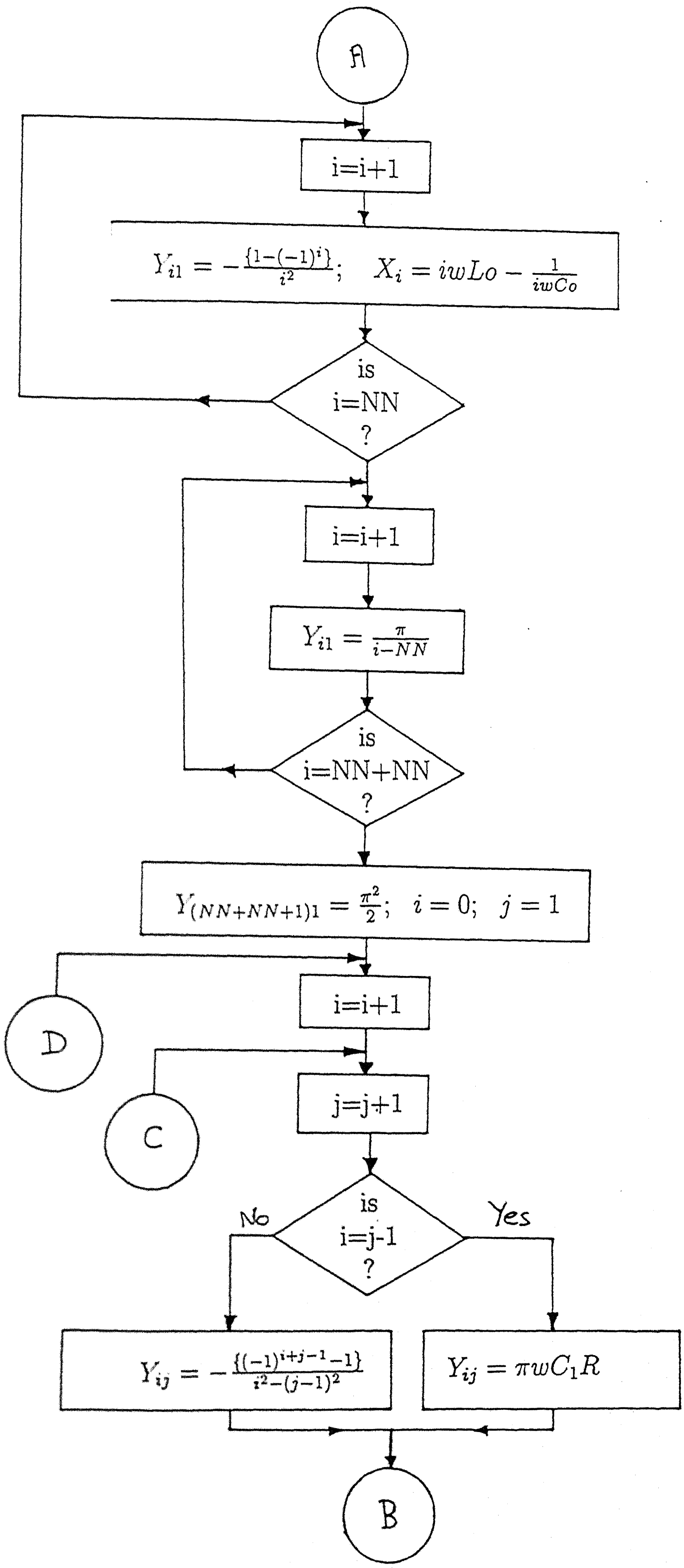
```
* PROGRAM FOR EXACT ANALYSIS OF CLASS-E AMPLIFIER (WITH DIODE),
* USING STATE EQUATIONS.
      double precision yt1(4),yt2(4), y(4),yy(4),yyy(4),yyyy(4)
      double precision rka(4),rkb(4),rkc(4),rkd(4)
      double precision vs,r,l1,lo,c1,co,t,tt,dt,f,tpd,d
      double precision err(4)
      open(unit=21,file='tt.i')
      open(unit=22,file='tt.vc')
      open(unit=23,file='tt.io')
      open(unit=24,file='tt.vco')
      open(unit=25,file='tt.is')
      vs=100.0d0
      f=50.00d3
      d=0.50d0
      l1=5.0d-3
      r=3.089d0
      c1=0.203d-6
      lo=90.13d-6
      co=0.13d-6
      tpd=1.0d0/f
      tt=tpd*d
      dt=1.0d-9
      m=0
      do 1 i=1,4
      y(i)=0.0d0
      yt2(i)=0.0d0
 1    continue
 1000 t=0.0d0
      n=5
c FOR  S - ON
      do 10 i=1,4
      yt1(i)=yt2(i)
 10   continue
 100  y(2)=0.0d0
      rka(1)=dt*(vs/l1)
      rka(3)=-dt*(y(4)/lo+y(3)*r/lo)
      rka(4)=dt*(y(3)/co)
      yy(1)=y(1)+rka(1)/2.0d0
      yy(3)=y(3)+rka(3)/2.0d0
      yy(4)=y(4)+rka(4)/2.0d0
      rkb(1)=dt*(vs/l1)
      rkb(3)=-dt*(yy(4)/lo+yy(3)*r/lo)
      rkb(4)=dt*(yy(3)/co)
      yyy(1)=yy(1)+rkb(1)/2.0d0
      yyy(3)=yy(3)+rkb(3)/2.0d0
      yyy(4)=yy(4)+rkb(4)/2.0d0
      rkc(1)=dt*(vs/l1)
      rkc(3)=-dt*(yyy(4)/lo+yyy(3)*r/lo)
      rkc(4)=dt*(yyy(3)/co)
      yyyy(1)=yyy(1)+rkc(1)
      yyyy(3)=yyy(3)+rkc(3)
      yyyy(4)=yyy(4)+rkc(4)
      rkd(1)=dt*(vs/l1)
      rkd(3)=-dt*(yyyy(4)/lo+yyyy(3)*r/lo)
      rkd(4)=dt*(yyyy(3)/co)
      y(1)=y(1)+(1.0d0/6.0d0)*(rka(1)+2.0d0*rkb(1)+2.0d0*rkc(1)+rkd(1))
      y(3)=y(3)+(1.0d0/6.0d0)*(rka(3)+2.0d0*rkb(3)+2.0d0*rkc(3)+rkd(3))
      y(4)=y(4)+(1.0d0/6.0d0)*(rka(4)+2.0d0*rkb(4)+2.0d0*rkc(4)+rkd(4))
      ss=y(1)-y(3)
      t=t+dt
      if(m.ne.1) go to 2
      if(n.eq.5) then
      n=0
      write(21,*)t,y(1)
      write(22,*)t,y(2)
      write(23,*)t,y(3)
```

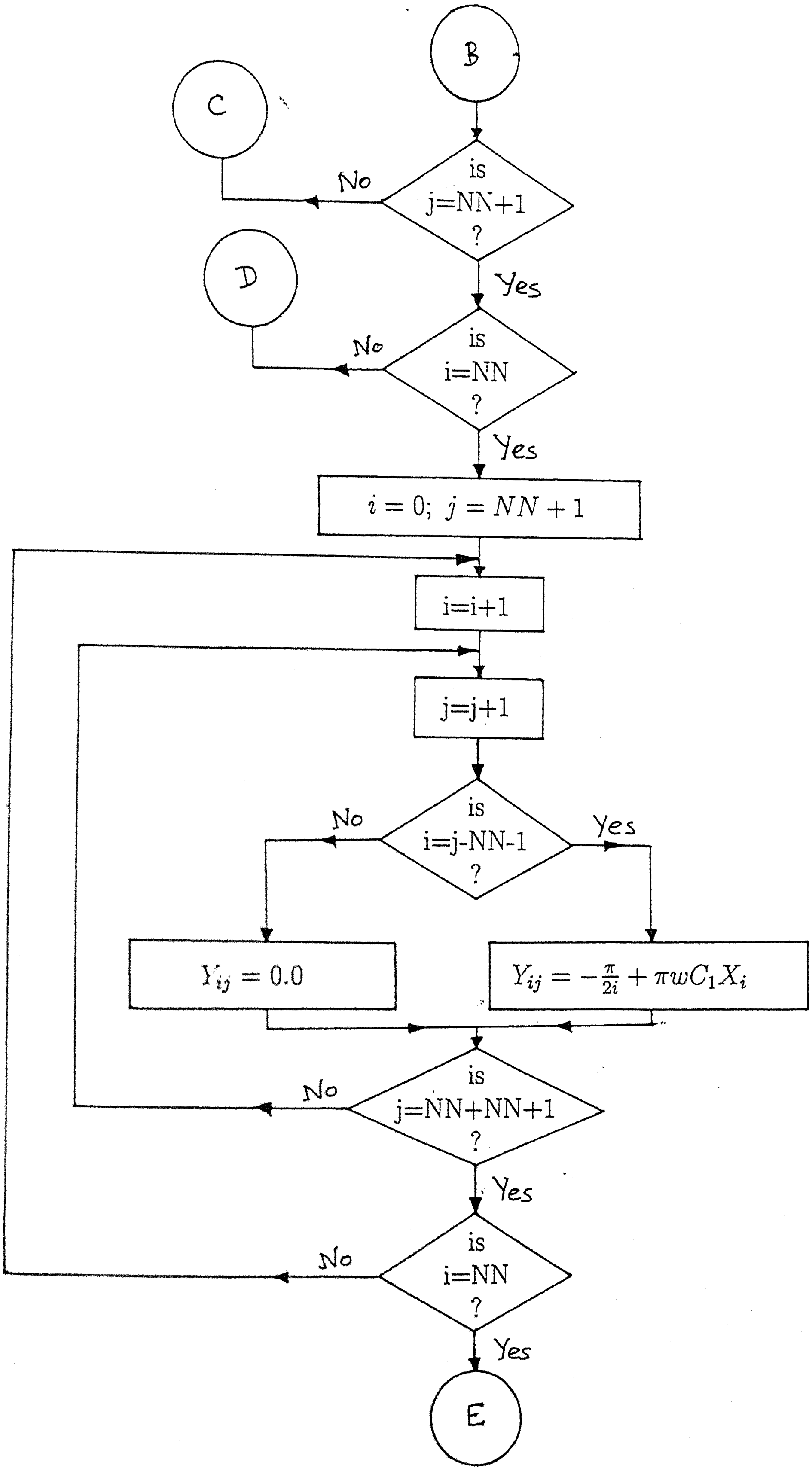
```
      write(24,*)t,y(4)
      write(25,*)t,ss
      endif
      n=n+1
  2   if(t.le.tt) go to 100
c FOR  S _ OFF
200   rka(1)=dt*(vs/l1-y(2)/l1)
      rka(2)=dt*(y(1)/c1-y(3)/c1)
      rka(3)=dt*((y(2)-y(4))/lo-y(3)*r/lo)
      rka(4)=dt*y(3)/co
      do 5 i=1,4
  5   yy(i)=y(i)+rka(i)/2.0d0
      rkb(1)=dt*(vs/l1-yy(2)/l1)
      rkb(2)=dt*(yy(1)/c1-yy(3)/c1)
      rkb(3)=dt*((yy(2)-yy(4))/lo-yy(3)*r/lo)
      rkb(4)=dt*yy(3)/co
      do 6 i=1,4
  6   yyy(i)=yy(i)+rkb(i)/2.0d0
      rkc(1)=dt*(vs/l1-yyy(2)/l1)
      rkc(2)=dt*(yyy(1)/c1-yyy(3)/c1)
      rkc(3)=dt*((yyy(2)-yyy(4))/lo-yyy(3)*r/lo)
      rkc(4)=dt*yyy(3)/co
      do 7 i=1,4
  7   yyyy(i)=yyy(i)+rkc(i)
      rkd(1)=dt*(vs/l1-yyyy(2)/l1)
      rkd(2)=dt*(yyyy(1)/c1-yyyy(3)/c1)
      rkd(3)=dt*((yyyy(2)-yyyy(4))/lo-yyyy(3)*r/lo)
      rkd(4)=dt*yyyy(3)/co
      do 8 i=1,4
  8   y(i)=y(i)+(1.0d0/6.0d0)*(rka(i)+2.0d0*rkb(i)+2.0d0*rkc(i)+rkd(i))
      ss=y(1)-y(3)
      t=t+dt
      if(y(2).lt.0.0d0) then
      y(2)=0.0d0
      endif
      if(m.ne.1) go to 3
      if(n.eq.5)then
      n=0
      write(21,*)t,y(1)
      write(22,*)t,y(2)
      write(23,*)t,y(3)
      write(24,*)t,y(4)
      write(25,*)t,ss
      endif
      n=n+1
  3   if(t.le.tpd) go to 200
      do 12 i=1,4
      yt2(i)=y(i)
      err(i)=abs(yt2(i)-yt1(i))
12    continue
      if(err(1) .gt. 0.1d-8)  go to 1000
      if(err(2) .gt. 0.1d-8)  go to 1000
      if(err(3) .gt. 0.1d-8)  go to 1000
      if(err(4) .gt. 0.1d-3)  go to 1000
      m=m+1
      if(m.lt.2) go to 1000
      close(21)
      close(22)
      close(23)
      close(24)
      close(25)
      stop
      end
```

Appendix B

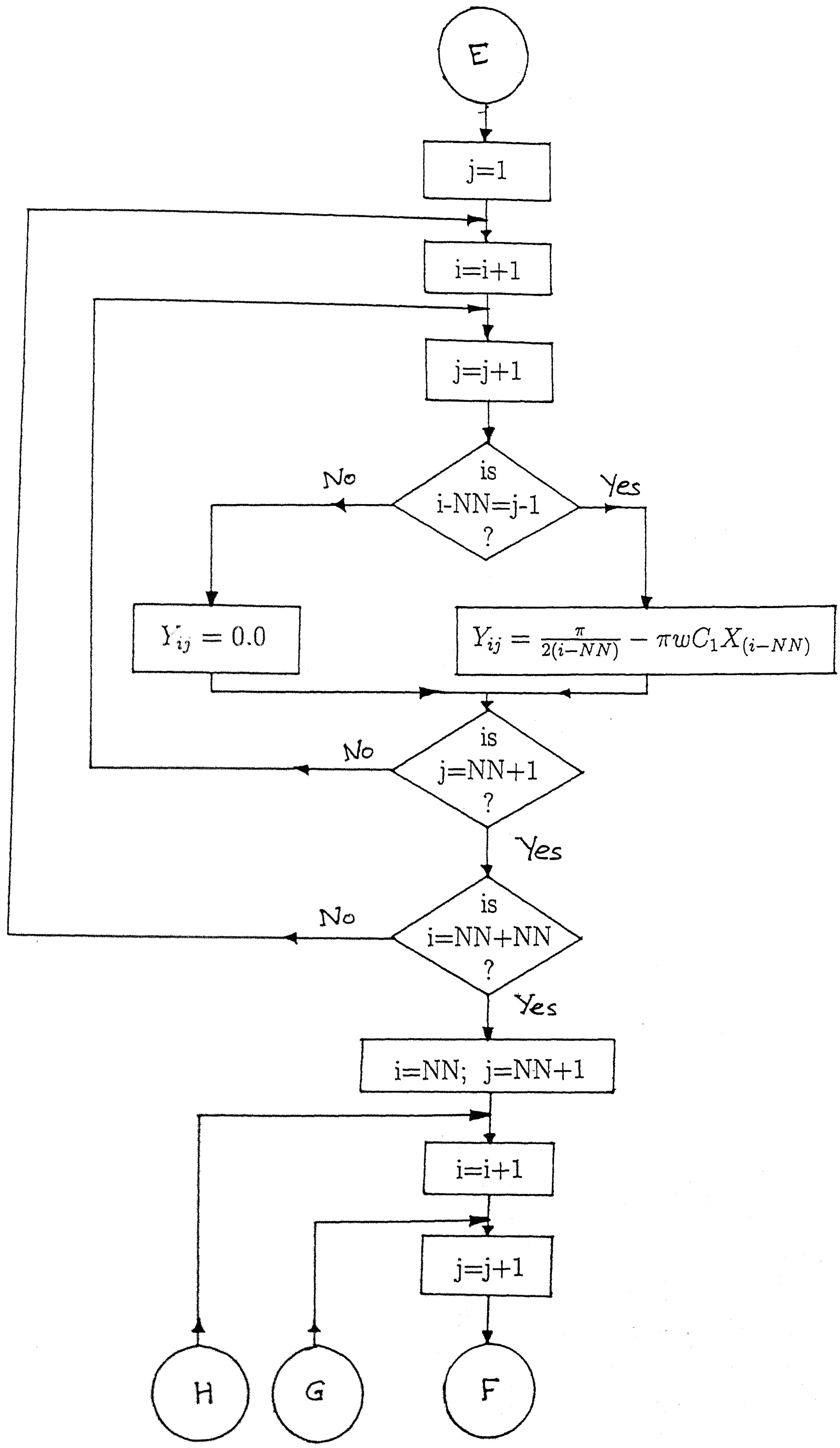
Flow-Chart For Analysis of Class-E Amplifier With Non-Sinusoidal Output Current

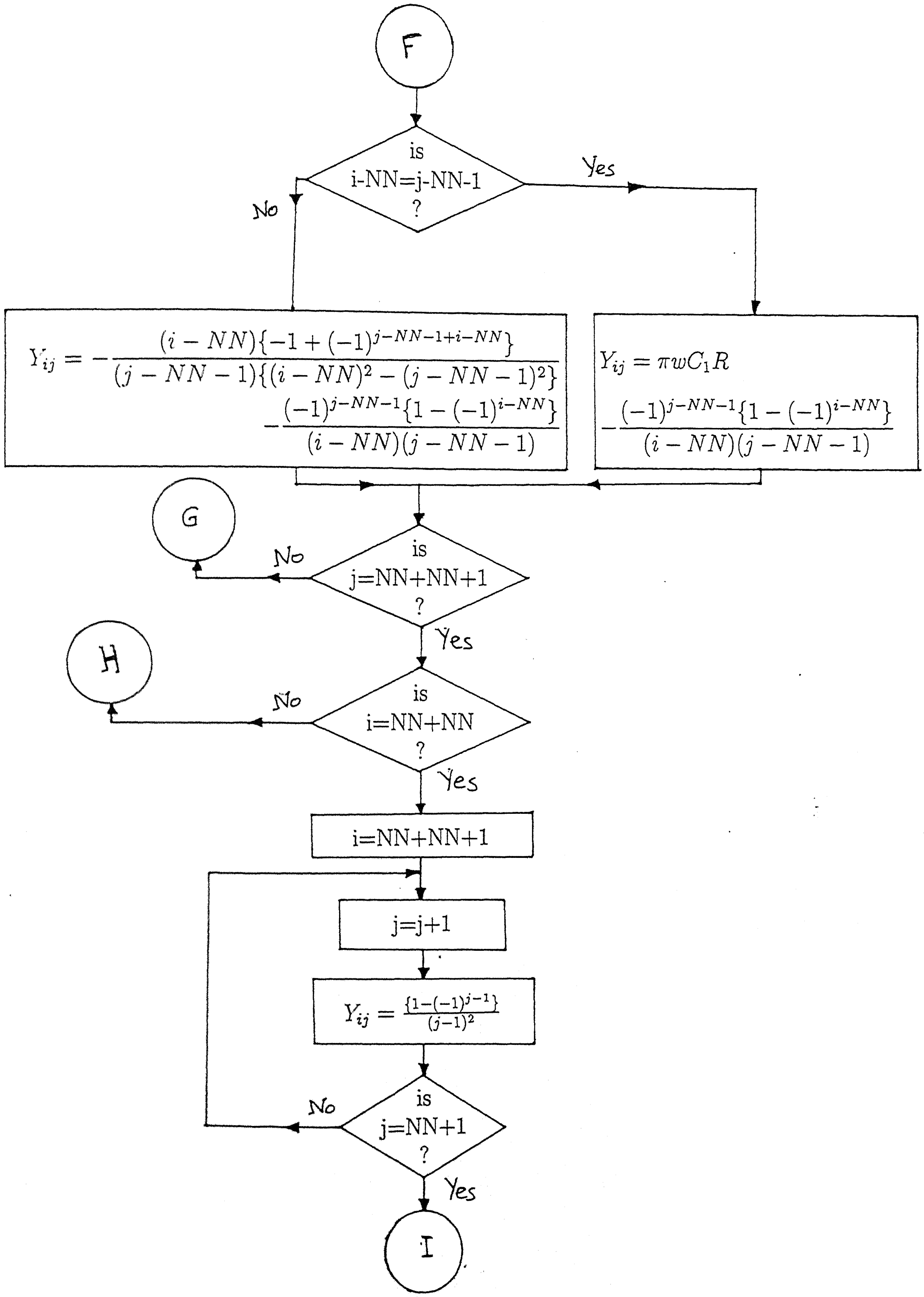


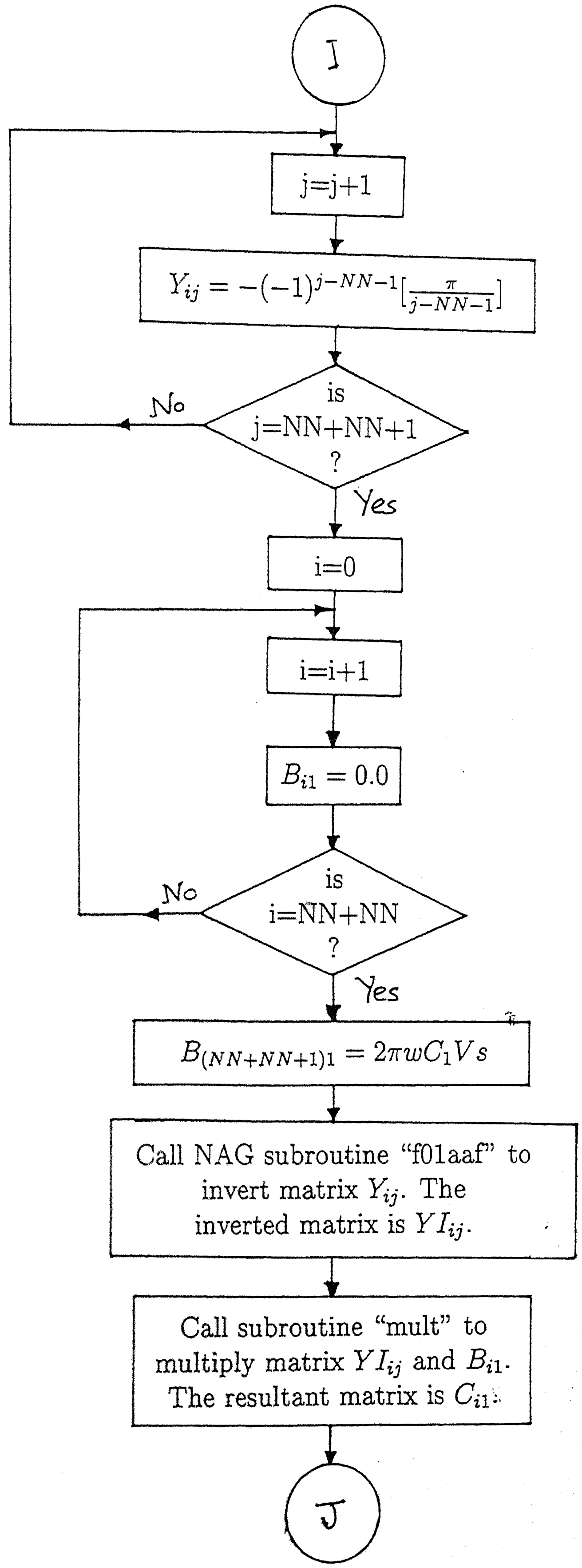

A
i=i+1
$Y_{i1} = -\frac{\{1-(-1)^i\}}{i^2}; \quad X_i = iwLo - \frac{1}{iwCo}$
is
i=NN
?
i=i+1
$Y_{i1} = \frac{\pi}{i-NN}$
is
i=NN+NN
?
$Y_{(NN+NN+1)1} = \frac{\pi^2}{2}; \quad i = 0; \quad j = 1$
i=i+1
D
j=j+1
C
is
i=j-1
?
No
Yes
$Y_{ij} = -\frac{\{(-1)^{i+j-1}-1\}}{i^2-(j-1)^2}$
$Y_{ij} = \pi w C_1 R$
B

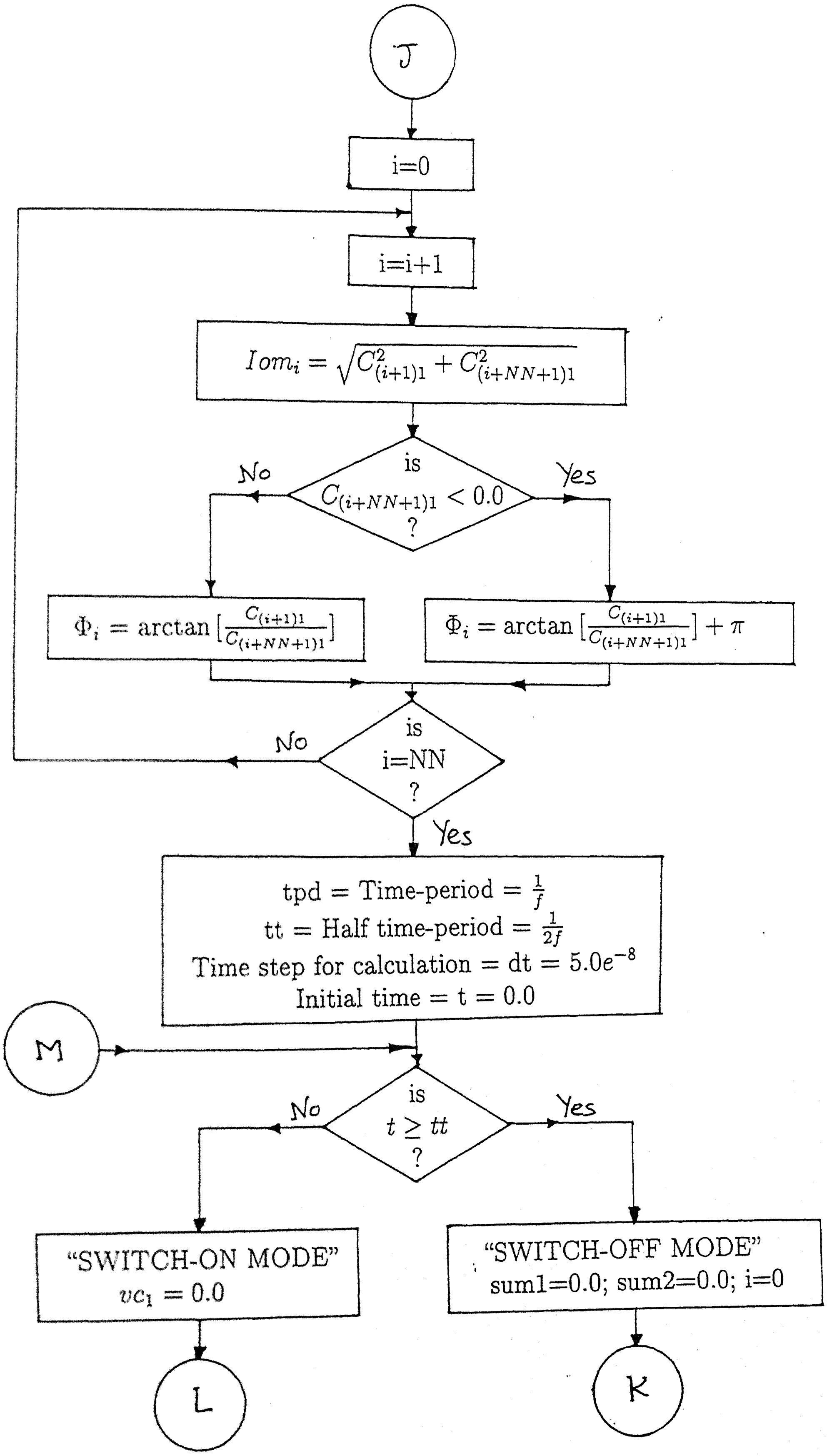


B
C
is j=NN+1 ?
No
Yes
D
is i=NN ?
No
Yes
$i = 0;\ j = NN + 1$
i=i+1
j=j+1
is i=j-NN-1 ?
No
Yes
$Y_{ij} = 0.0$
$Y_{ij} = -\frac{\pi}{2i} + \pi w C_1 X_i$
is j=NN+NN+1 ?
No
Yes
is i=NN ?
No
Yes
E

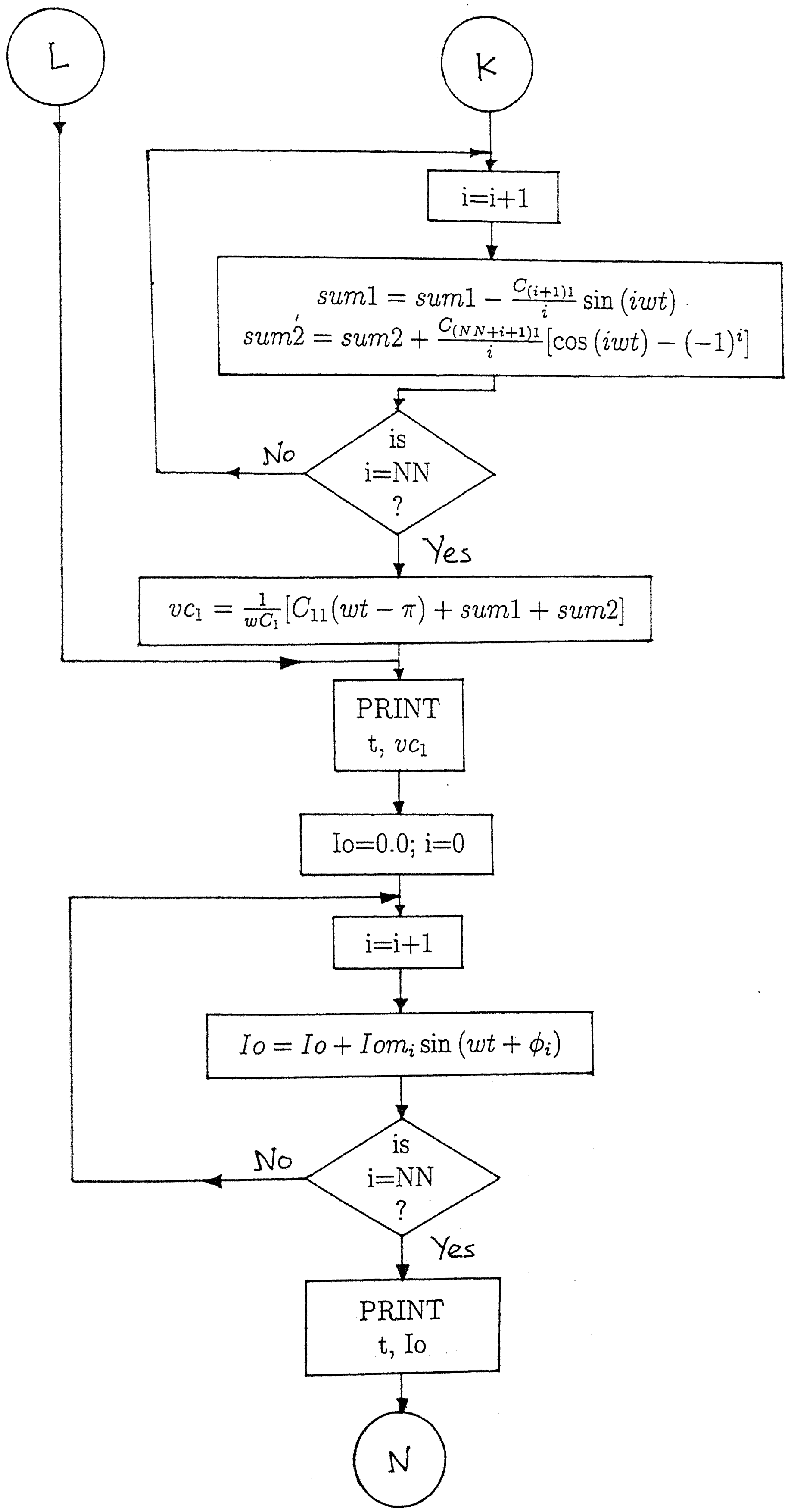

E
j=1
i=i+1
j=j+1
is
i-NN=j-1
?
No
Yes
$Y_{ij} = 0.0$
$Y_{ij} = \frac{\pi}{2(i-NN)} - \pi w C_1 X_{(i-NN)}$
is
j=NN+1
?
No
Yes
is
i=NN+NN
?
No
Yes
i=NN; j=NN+1
i=i+1
j=j+1
H
G
F

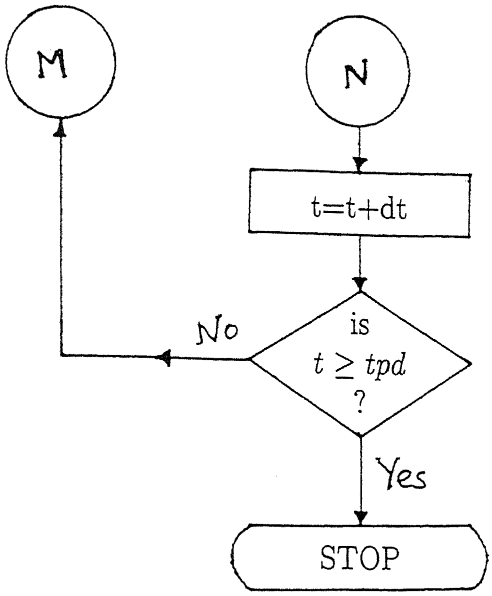

F
is
i-NN=j-NN-1
?
Yes
No
$Y_{ij} = -\frac{(i-NN)\{-1+(-1)^{j-NN-1+i-NN}\}}{(j-NN-1)\{(i-NN)^2-(j-NN-1)^2\}} - \frac{(-1)^{j-NN-1}\{1-(-1)^{i-NN}\}}{(i-NN)(j-NN-1)}$
$Y_{ij} = \pi w C_1 R - \frac{(-1)^{j-NN-1}\{1-(-1)^{i-NN}\}}{(i-NN)(j-NN-1)}$
G
No
is
j=NN+NN+1
?
Yes
H
No
is
i=NN+NN
?
Yes
i=NN+NN+1
j=j+1
$Y_{ij} = \frac{\{1-(-1)^{j-1}\}}{(j-1)^2}$
No
is
j=NN+1
?
Yes
I


I
j=j+1
$Y_{ij} = -(-1)^{j-NN-1}[\frac{\pi}{j-NN-1}]$
is j=NN+NN+1 ?
No
Yes
i=0
i=i+1
$B_{i1} = 0.0$
is i=NN+NN ?
No
Yes
$B_{(NN+NN+1)1} = 2\pi w C_1 V s$
Call NAG subroutine "f01aaf" to invert matrix Y_{ij}. The inverted matrix is YI_{ij}.
Call subroutine "mult" to multiply matrix YI_{ij} and B_{i1}. The resultant matrix is C_{i1}.
J

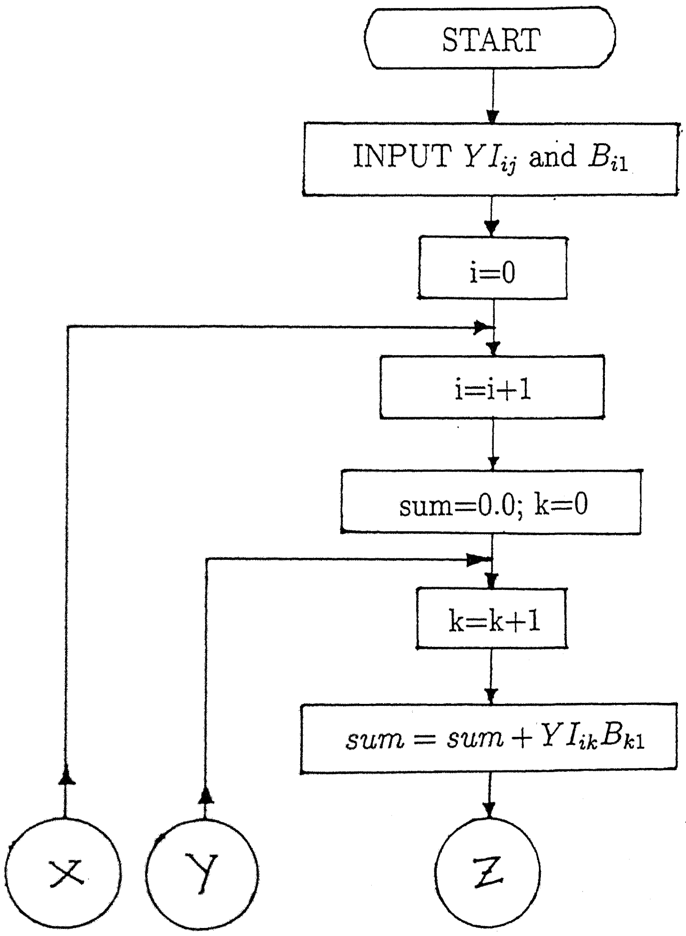


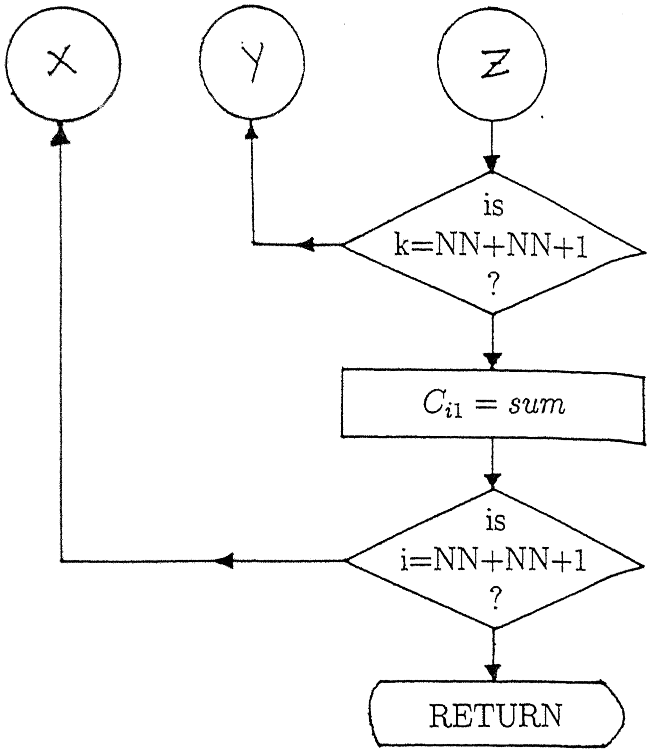
J
i=0
i=i+1
$Iom_i = \sqrt{C^2_{(i+1)1} + C^2_{(i+NN+1)1}}$
is $C_{(i+NN+1)1} < 0.0$?
No
Yes
$\Phi_i = \arctan\left[\frac{C_{(i+1)1}}{C_{(i+NN+1)1}}\right]$
$\Phi_i = \arctan\left[\frac{C_{(i+1)1}}{C_{(i+NN+1)1}}\right] + \pi$
is i=NN ?
No
Yes
tpd = Time-period = $\frac{1}{f}$
tt = Half time-period = $\frac{1}{2f}$
Time step for calculation = dt = $5.0e^{-8}$
Initial time = t = 0.0
M
is $t \geq tt$?
No
Yes
"SWITCH-ON MODE"
$vc_1 = 0.0$
"SWITCH-OFF MODE"
sum1=0.0; sum2=0.0; i=0
L
K


L
K
i=i+1
$sum1 = sum1 - \frac{C_{(i+1)1}}{i}\sin(iwt)$
$sum2 = sum2 + \frac{C_{(NN+i+1)1}}{i}[\cos(iwt) - (-1)^i]$
No
is i=NN ?
Yes
$vc_1 = \frac{1}{wC_1}[C_{11}(wt - \pi) + sum1 + sum2]$
PRINT t, vc_1
Io=0.0; i=0
i=i+1
$Io = Io + Iom_i \sin(wt + \phi_i)$
No
is i=NN ?
Yes
PRINT t, Io
N



Flow-Chart For Subroutine "mult"




X
Y
Z
is
k=NN+NN+1
?
$C_{i1} = sum$
is
i=NN+NN+1
?
RETURN

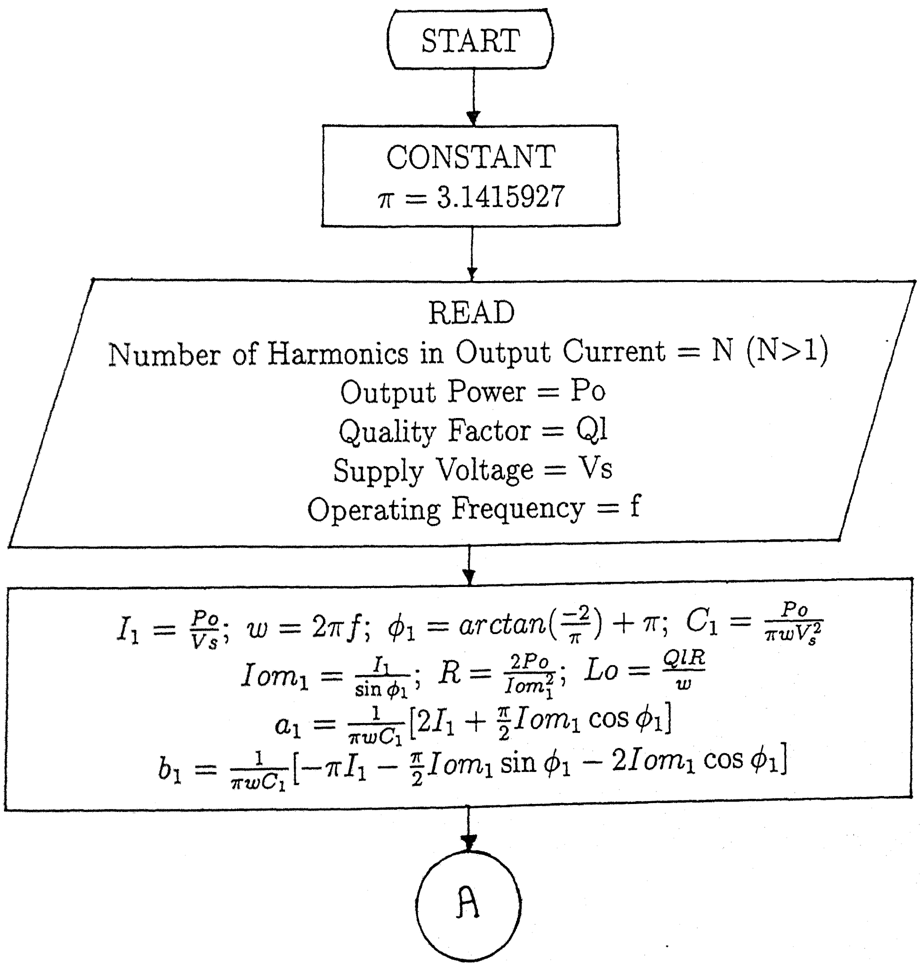
```
c PROGRAM FOR ANALYSIS OF CLASS-E AMPLIFIER WITH NON-SINUSOIDAL
c OUTPUT CURRENT.
      parameter (nn=10,nmax=(nn+nn+1),ia=nmax,ix=nmax,n=nmax)
      double precision vs,f,c1,lo,co,pi,r,t,tt,tpd,dt
      double precision x(nn),w,vc1,v1,sum1,sum2,q,q1,qq,io
      double precision y(n,n),yi(n,n),b(n,1),a(n,1),c(n,1)
      double precision iom(nn),phi(nn)
      external f01aaf
      open (unit=21,file='ht.vc')
      open (unit=22,file='y1')
      open (unit=23,file='b1')
      open (unit=24,file='yi1')
      open (unit=25,file='c1')
      open (unit=29,file='ht.io')
      pi=3.14159270d0
      vs=100.0d0
      c1=0.2069d-6
      lo=367.20280d-6
      co=0.028420d-6
      r=2.884d0
      f=50.00d3
      w=2.0d0*pi*f
      do 1 i=1,nn
      x(i)=i*w*lo-(1.0d0/(i*w*co))
 1    y(i,1)=-(1.0d0/(i**2))*(1.0d0-(-1)**(i))
      do 2 i=(nn+1),(nn+nn)
 2    y(i,1)=pi/(i-nn)
      y((nn+nn+1),1)=(pi**2)/2.0d0
      do 3 i=1,nn
      do 3 j=2,(nn+1)
      if(i.eq.(j-1))then
      y(i,j)=pi*w*c1*r
      else
      y(i,j)=-(((-1)**(i+j-1))-1.0d0)/(i**2-(j-1)**2)
      endif
 3    continue
      do 4 i=1,nn
      do 4 j=(nn+2),(nn+nn+1)
      if(i.eq.(j-nn-1))then
      y(i,j)=-(pi/(2.0d0*i))+pi*w*c1*x(i)
      else
      y(i,j)=0.0d0
      endif
 4    continue
      do 5 i=(nn+1),(nn+nn)
      do 5 j=2,(nn+1)
      if((i-nn).eq.(j-1))then
      y(i,j)= (pi/(2.0d0*(i-nn)))-pi*w*c1*x(i-nn)
      else
      y(i,j)=0.0d0
      endif
 5    continue
      do 6 i=(nn+1),(nn+nn)
      do 6 j=(nn+2),(nn+nn+1)
      if((i-nn).eq.(j-nn-1))then
      qq= pi*w*c1*r
      y(i,j)=qq-((-1)**(j-nn-1))*(1.0d0-(-1)**(i-nn))/((i-nn)*(j-nn-1))
      else
      q=((-1)**(j-nn-1))*(1.0d0-((-1)**(i-nn)))/((i-nn)*(j-nn-1))
      q1=-1.0d0+(-1)**(j-nn-1+i-nn)
      y(i,j)=-q1*(i-nn)/((j-nn-1)*(((i-nn)**2)-((j-nn-1)**2)))-q
      endif
 6    continue
      do 7 i=(nn+nn+1),(nn+nn+1)
      do 7 j=2,(nn+1)
 7    y(i,j)=(1.0d0-(-1)**(j-1))/((j-1)**2)
```

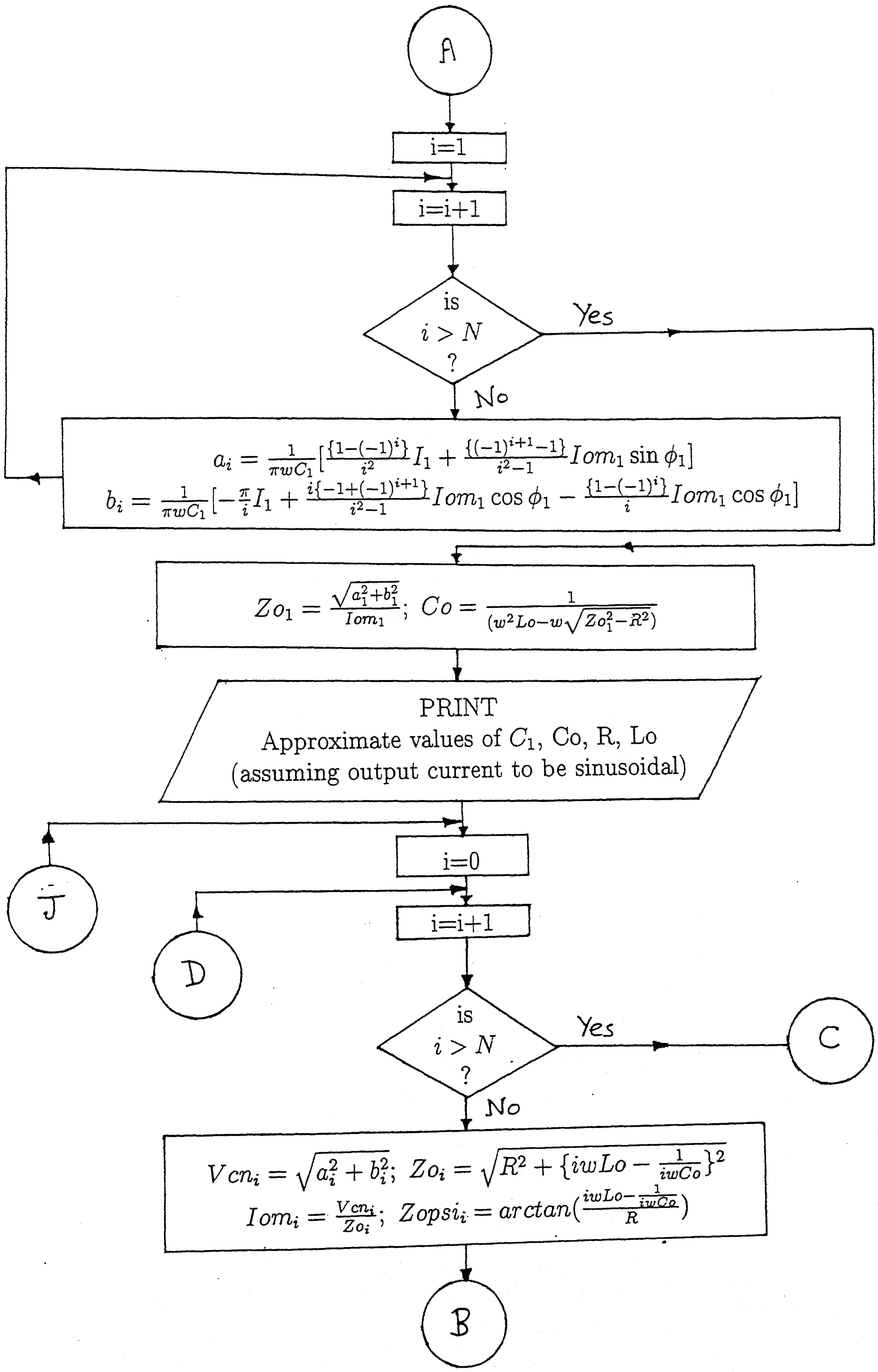
```
        do 8 i=(nn+nn+1),(nn+nn+1)
        do 8 j=(nn+2),(nn+nn+1)
8       y(i,j)=-((-1)**(j-nn-1))*pi/(j-nn-1)
        do 21 i=1,(nn+nn)
21       b(i,1)=0.0d0
        b((nn+nn+1),1)=2.0d0*pi*w*c1*vs
        do 101 i=1,n
        write(22,*)(y(i,j),j=1,n)
101     continue
        write(23,*)(b(i,1),i=1,n)
        call f01aaf(y,ia,n,yi,ix,a,ifail)
        write(24,*)((yi(i,j),i=1,n),j=1,n)
        call mult(n,n,1,yi,b,c)
        write(25,*)(c(i,1),i=1,n)
        do 111 i=1,nn
        iom(i)=sqrt(c(i+1,1)**2+c(i+nn+1,1)**2)
        if(c(i+nn+1,1).lt.0.0d0) then
        phi(i)=datan(c(i+1,1)/c(i+nn+1,1))+pi
        else
        phi(i)=datan(c(i+1,1)/c(i+nn+1,1))
        endif
111     continue
        tpd=1.0d0/f
        tt=1.0d0/(2.0d0*f)
        dt=5.0d-3
        t=0.0d0
20      if(t.ge.tt) then
        v1=c(1,1)*(w*t-pi)
        sum1=0.0d0
        sum2=0.0d0
        do 233 i=1,nn
        sum1=-(c((i+1),1)/i)*sin(i*w*t)+sum1
        sum2=(c((nn+i+1),1)/i)*(cos(i*w*t)-((-1)**i))+sum2
233     continue
        vc1=(1.0d0/(w*c1))*(v1+sum1+sum2)
        write(21,*)t,vc1
        else
        vc1=0.0d0
        write(21,*)t,vc1
        endif
        io=0.0d0
        do 555 i=1,nn
        io=io+iom(i)*dsin(w*t+phi(i))
555     continue
        write(29,*)t,io
        t=t+dt
        if(t.le.(tpd+dt)) go to 20
        close(22)
        close(23)
        close(24)
        close(25)
        close(29)
        stop
        end
c       ----------------------------
        subroutine mult(n,m,iip,aa,bb,cc)
        double precision aa(n,n),bb(n,1),cc(n,1)
        double precision sum
        do 5 i=1,n
        do 5 j=1,iip
        sum=0.0
        do 6 k=1,m
6       sum=sum+aa(i,k)*bb(k,j)
5       cc(i,j)=sum
        return
        end
```

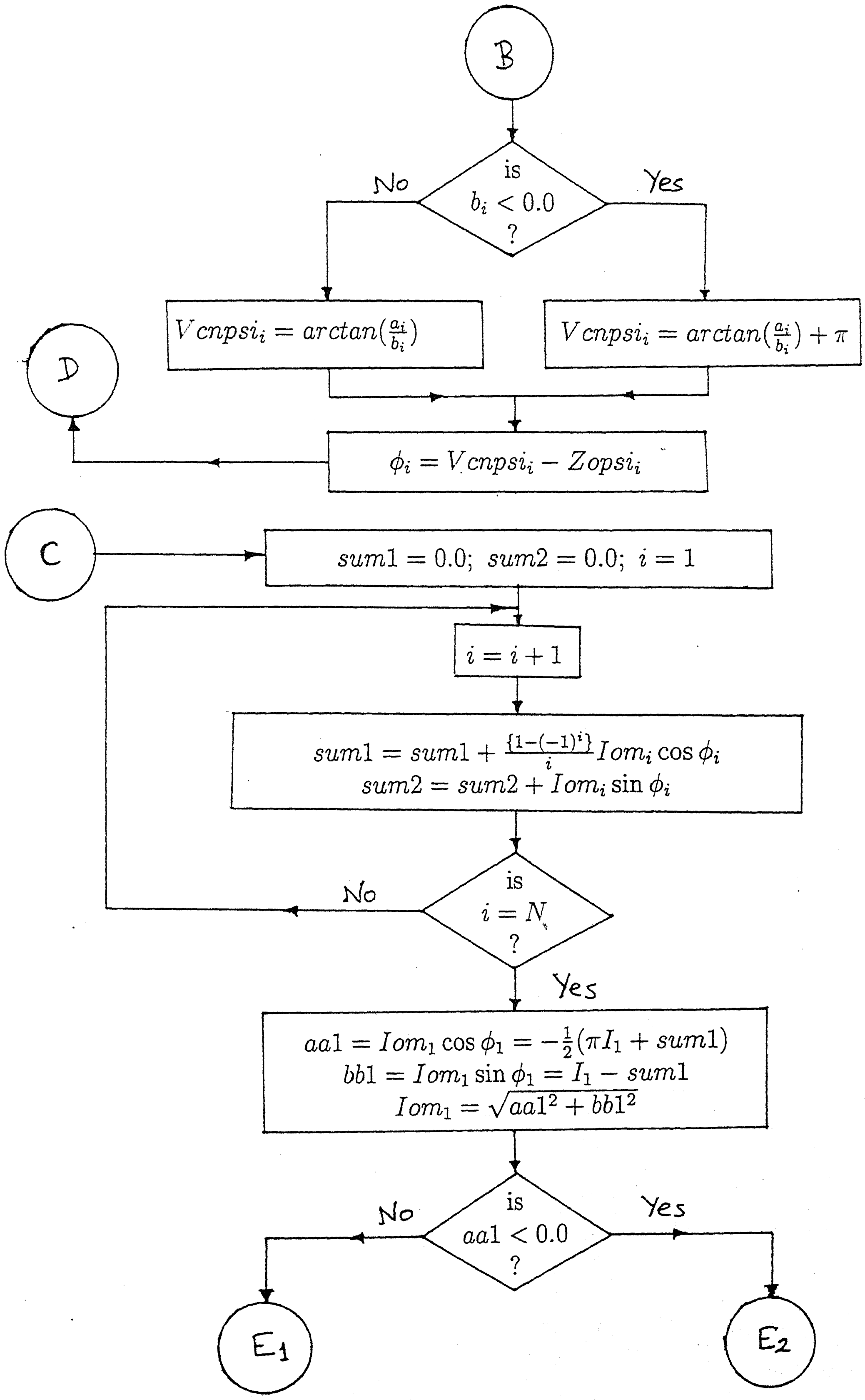
Appendix C

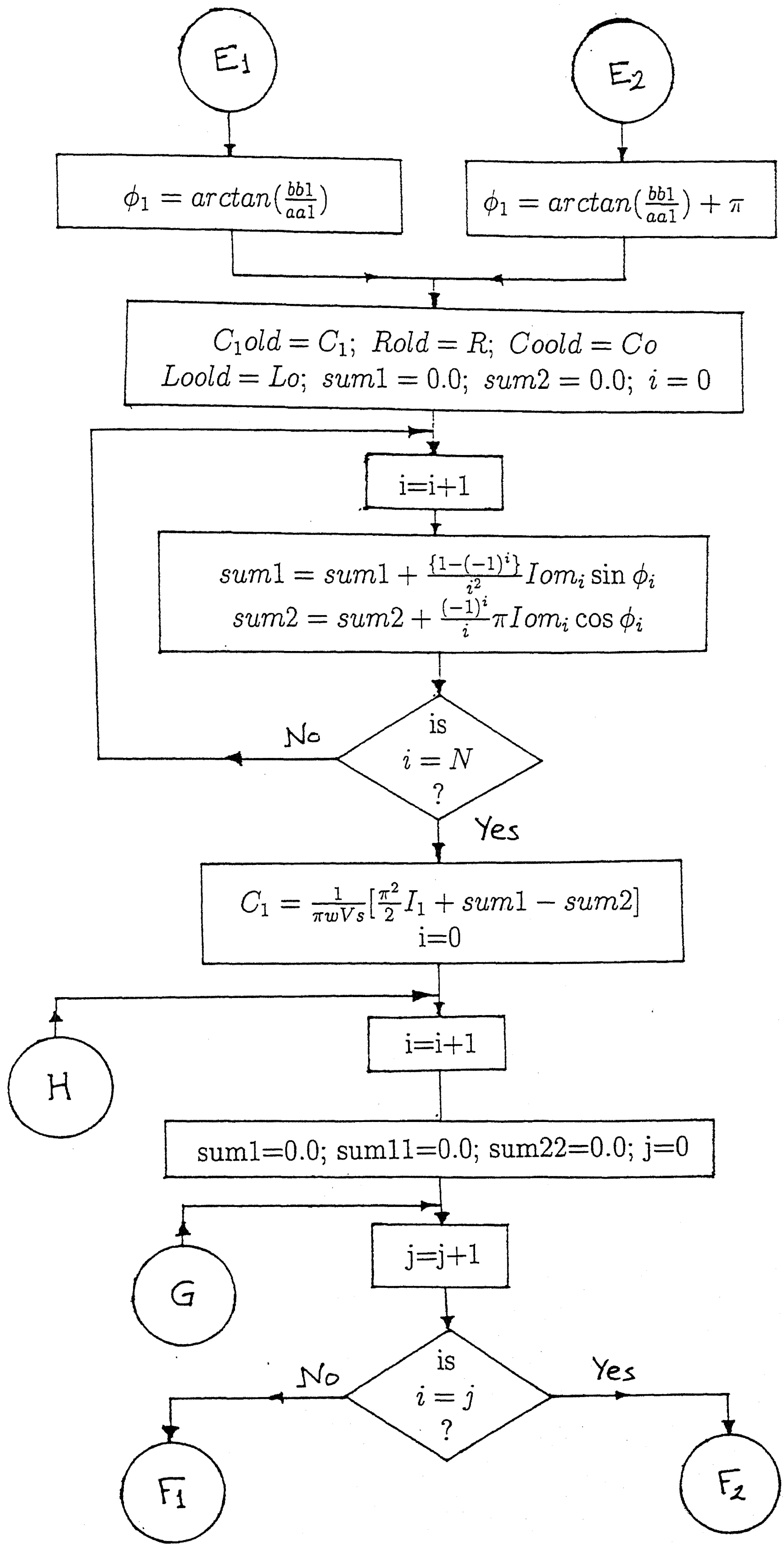
Flow-Chart For Accurate Design of Class-E Amplifier

C.1 With Vs, Po, Ql and f Constant

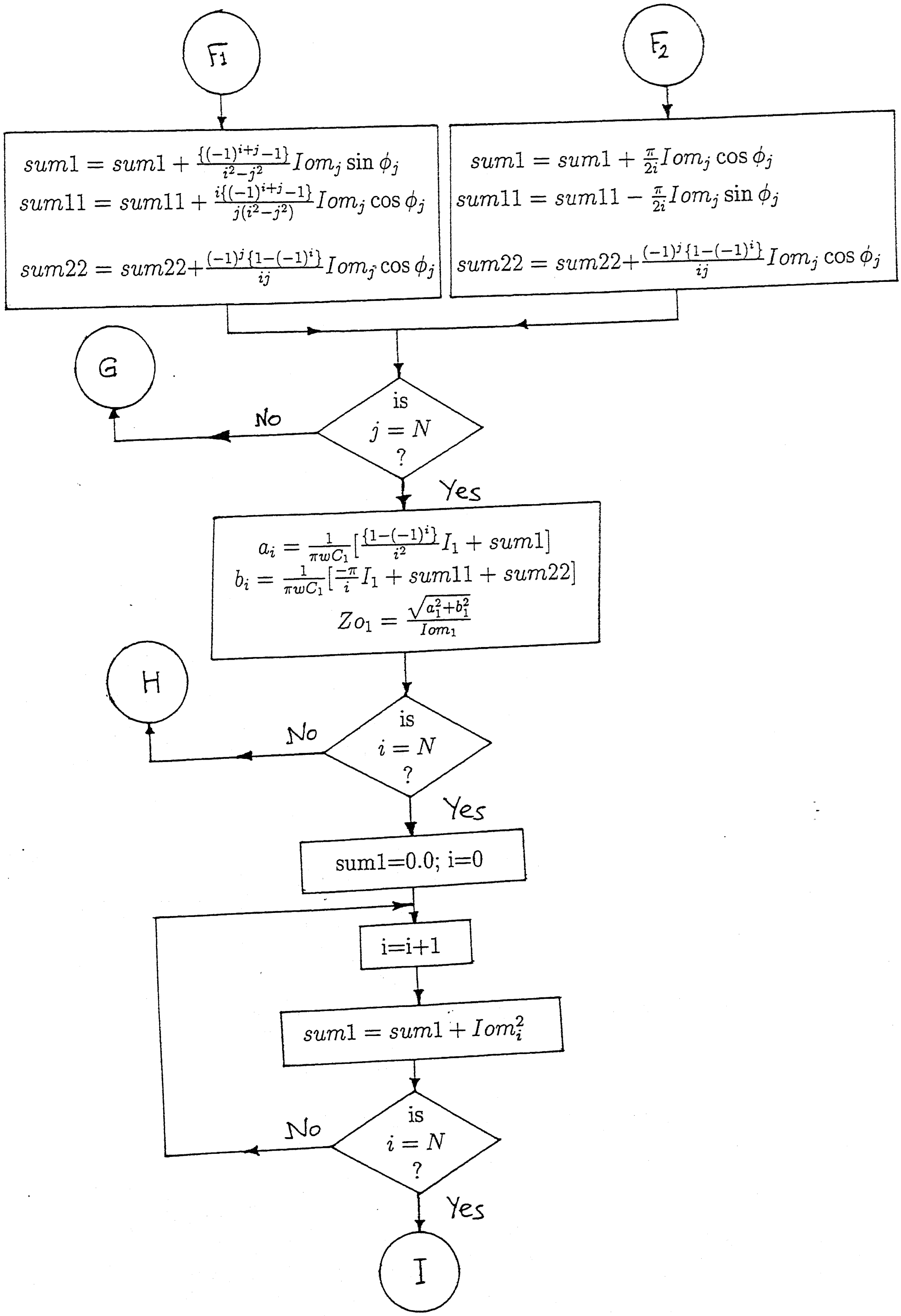


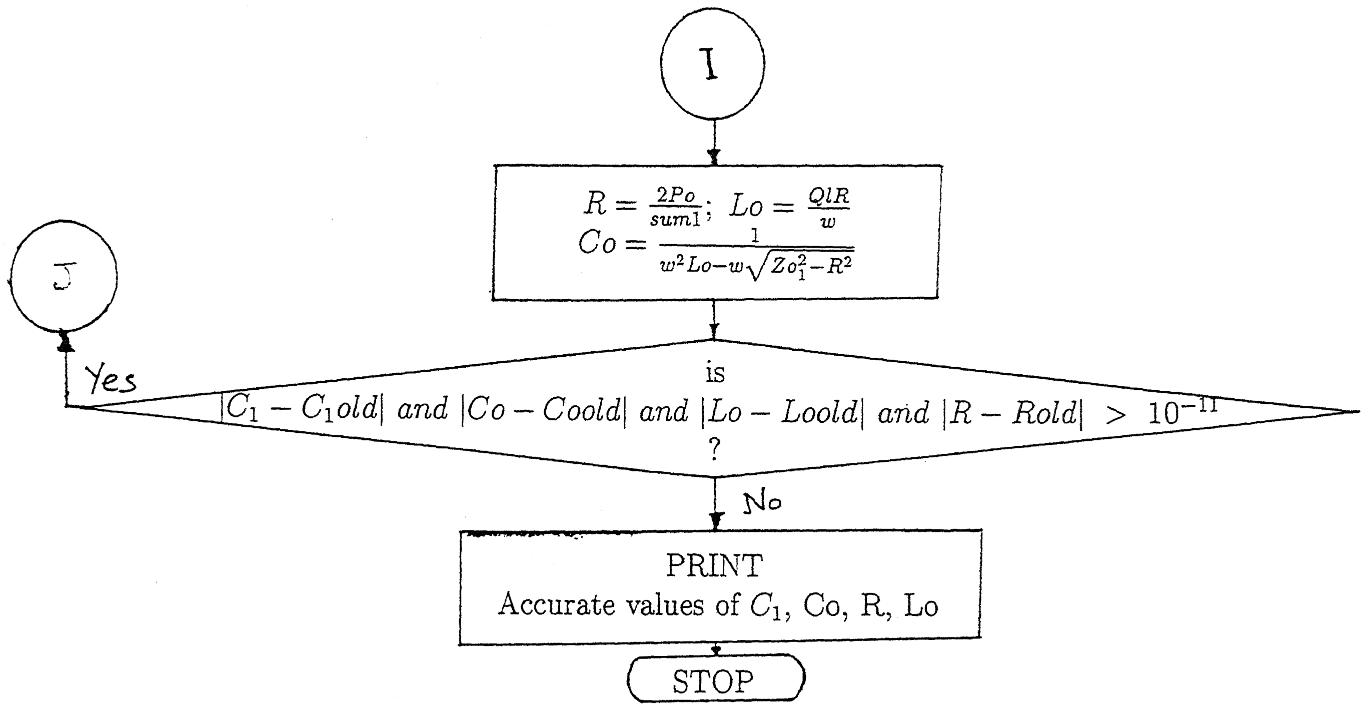

A
i=1
i=i+1
is
$i > N$
?
Yes
No
$a_i = \frac{1}{\pi w C_1}[\frac{\{1-(-1)^i\}}{i^2} I_1 + \frac{\{(-1)^{i+1}-1\}}{i^2-1} Iom_1 \sin\phi_1]$
$b_i = \frac{1}{\pi w C_1}[-\frac{\pi}{i} I_1 + \frac{i\{-1+(-1)^{i+1}\}}{i^2-1} Iom_1 \cos\phi_1 - \frac{\{1-(-1)^i\}}{i} Iom_1 \cos\phi_1]$
$Zo_1 = \frac{\sqrt{a_1^2+b_1^2}}{Iom_1};\ Co = \frac{1}{(w^2 Lo - w\sqrt{Zo_1^2-R^2})}$
PRINT
Approximate values of C_1, Co, R, Lo
(assuming output current to be sinusoidal)
J
i=0
D
i=i+1
is
$i > N$
?
Yes
C
No
$Vcn_i = \sqrt{a_i^2 + b_i^2};\ Zo_i = \sqrt{R^2 + \{iwLo - \frac{1}{iwCo}\}^2}$
$Iom_i = \frac{Vcn_i}{Zo_i};\ Zopsi_i = arctan(\frac{iwLo - \frac{1}{iwCo}}{R})$
B


B
is
$b_i < 0.0$
?
No
Yes
$Vcnpsi_i = arctan(\frac{a_i}{b_i})$
$Vcnpsi_i = arctan(\frac{a_i}{b_i}) + \pi$
$\phi_i = Vcnpsi_i - Zopsi_i$
D
C
$sum1 = 0.0;\ sum2 = 0.0;\ i = 1$
$i = i + 1$
$sum1 = sum1 + \frac{\{1-(-1)^i\}}{i} Iom_i \cos\phi_i$
$sum2 = sum2 + Iom_i \sin\phi_i$
is
$i = N$
?
No
Yes
$aa1 = Iom_1 \cos\phi_1 = -\frac{1}{2}(\pi I_1 + sum1)$
$bb1 = Iom_1 \sin\phi_1 = I_1 - sum1$
$Iom_1 = \sqrt{aa1^2 + bb1^2}$
is
$aa1 < 0.0$
?
No
Yes
E_1
E_2



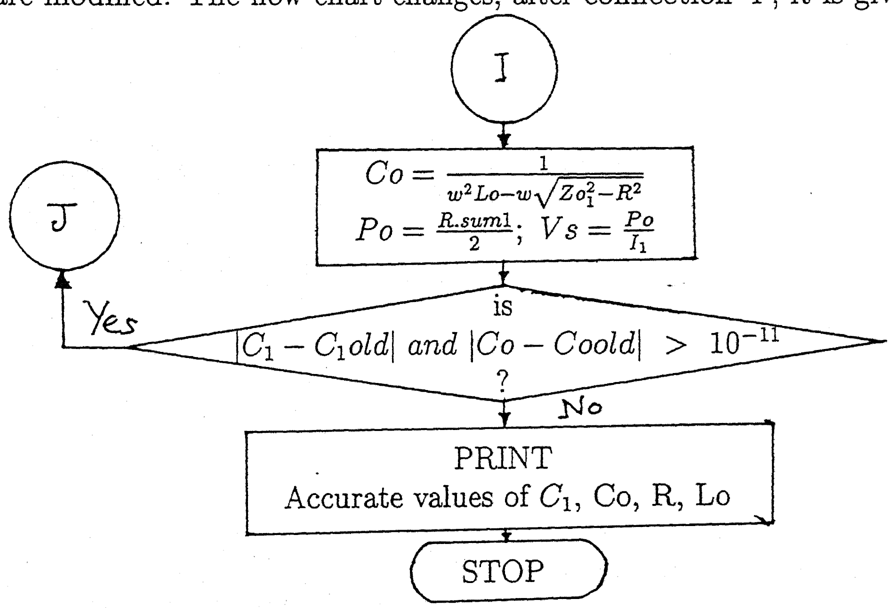
E_1
E_2
$\phi_1 = arctan(\frac{bb1}{aa1})$
$\phi_1 = arctan(\frac{bb1}{aa1}) + \pi$
$C_1old = C_1;\ Rold = R;\ Coold = Co$
$Loold = Lo;\ sum1 = 0.0;\ sum2 = 0.0;\ i = 0$
i=i+1
$sum1 = sum1 + \frac{\{1-(-1)^i\}}{i^2} Iom_i \sin\phi_i$
$sum2 = sum2 + \frac{(-1)^i}{i}\pi Iom_i \cos\phi_i$
No
is $i = N$?
Yes
$C_1 = \frac{1}{\pi w Vs}[\frac{\pi^2}{2} I_1 + sum1 - sum2]$
i=0
H
i=i+1
sum1=0.0; sum11=0.0; sum22=0.0; j=0
G
j=j+1
No
is $i = j$?
Yes
F_1
F_2


F1
F2
$sum1 = sum1 + \frac{\{(-1)^{i+j}-1\}}{i^2-j^2} Iom_j \sin\phi_j$
$sum11 = sum11 + \frac{i\{(-1)^{i+j}-1\}}{j(i^2-j^2)} Iom_j \cos\phi_j$
$sum22 = sum22 + \frac{(-1)^j\{1-(-1)^i\}}{ij} Iom_j \cos\phi_j$
$sum1 = sum1 + \frac{\pi}{2i} Iom_j \cos\phi_j$
$sum11 = sum11 - \frac{\pi}{2i} Iom_j \sin\phi_j$
$sum22 = sum22 + \frac{(-1)^j\{1-(-1)^i\}}{ij} Iom_j \cos\phi_j$
G
No
is $j = N$?
Yes
$a_i = \frac{1}{\pi w C_1}[\frac{\{1-(-1)^i\}}{i^2} I_1 + sum1]$
$b_i = \frac{1}{\pi w C_1}[\frac{-\pi}{i} I_1 + sum11 + sum22]$
$Zo_1 = \frac{\sqrt{a_1^2+b_1^2}}{Iom_1}$
H
No
is $i = N$?
Yes
sum1=0.0; i=0
i=i+1
$sum1 = sum1 + Iom_i^2$
No
is $i = N$?
Yes
I



C.2 With R, Ql and f Constant

The flow-chart of this section will be same as the flow-chart of previous section, except the last computation block. The calculated values of R and Lo, by approximate design method, are retained. Only values of C_1 and Co are modified. The flow-chart changes, after connection 'I'; it is given below.



```
C PROGRAM FOR Accurate Design With Vs, Po, Ql and f Constant; (Put n>1)
      parameter(n=10)
      double precision i1,po,vs,f,c1,c1old,lo,co,pi,r
      double precision w,ac,a(n),b(n),vcn(n),vcnpsi(n)
      double precision zo(n),zopsi(n),c,loold,rold,coold
      double precision aa1,bb1,aac
      double precision phi(n),iom(n),sum1,sum2,sum3,sum11,sum22
      pi=3.14159270d0
      po=2000.0d0
      ql=40.0d0
      vs=100.0d0
      f=50.00d3
      i1=po/vs
      w=2.0d0*pi*f
      phi(1)=datan(-2.0d0/pi)+pi
      c1=po/(pi*w*(vs**2))
      iom(1)=i1/dsin(phi(1))
      r=po*2.0d0/(iom(1)*iom(1))
      lo=ql*r/w
      ac=1.0d0/(pi*w*c1)
      a(1)=ac*(2.0d0*i1+0.5d0*pi*iom(1)*dcos(phi(1)))
      sum1=2.0d0*iom(1)*dcos(phi(1))
      b(1)=ac*(-pi*i1-0.5d0*pi*iom(1)*dsin(phi(1))-sum1)
      do 1 i=2,n
      c=dfloat(i)
      sum1=(((-1.0d0)**(c+1)-1.0d0)*iom(1)*dsin(phi(1)))/(c**2-1.0d0)
      a(i)=ac*(i1*(1.0d0-(-1.0d0)**c)/(c**2)+sum1)
 1    continue
      do 2 i=2,n
      c=dfloat(i)
      sum1=-(1.0d0-(-1)**c)*iom(1)*dcos(phi(1))/c
      sum2=c*(-1.0d0+(-1.0)**(c+1.0))*iom(1)*dcos(phi(1))/(c**2-1.0)
      b(i)=ac*(-(pi/c)*i1+sum2+sum1)
 2    continue
      zo(1)=sqrt(a(1)**2+b(1)**2)/(iom(1))
      co=1.0d0/(w*w*lo-w*(sqrt(zo(1)*zo(1)-r*r)))
      write(*,*)'Approximate values of c1,co,r,lo=',c1,co,r,lo
1100  do 3 i=1,n
      c=dfloat(i)
      vcn(i)=sqrt(a(i)**2+b(i)**2)
      zo(i)=sqrt(r**2+(c*w*lo-(1.0d0/(c*w*co)))**2)
      zopsi(i)=atan((c*w*lo-(1.0d0/(c*w*co)))/r)
      iom(i)=vcn(i)/zo(i)
      if(b(i).lt.0.0d0)then
      vcnpsi(i)=atan(a(i)/b(i))+pi
      else
      vcnpsi(i)=atan(a(i)/b(i))
      endif
      phi(i)=vcnpsi(i)-zopsi(i)
 3    continue
      sum1=0.0d0
      sum2=0.0d0
      do 4 i=2,n
      c=dfloat(i)
      sum1=sum1+(1.0d0-(-1.0d0)**i)*iom(i)*dcos(phi(i))/c
      sum2=sum2+iom(i)*dsin(phi(i))
 4    continue
      aa1=-0.5d0*(pi*i1+sum1)
      bb1=i1-sum2
      iom(1)=sqrt(aa1**2+bb1**2)
      if(aa1.lt.0.0d0)then
      phi(1)=atan(bb1/aa1)+pi
      else
      phi(1)=atan(bb1/aa1)
      endif
      c1old=c1
```

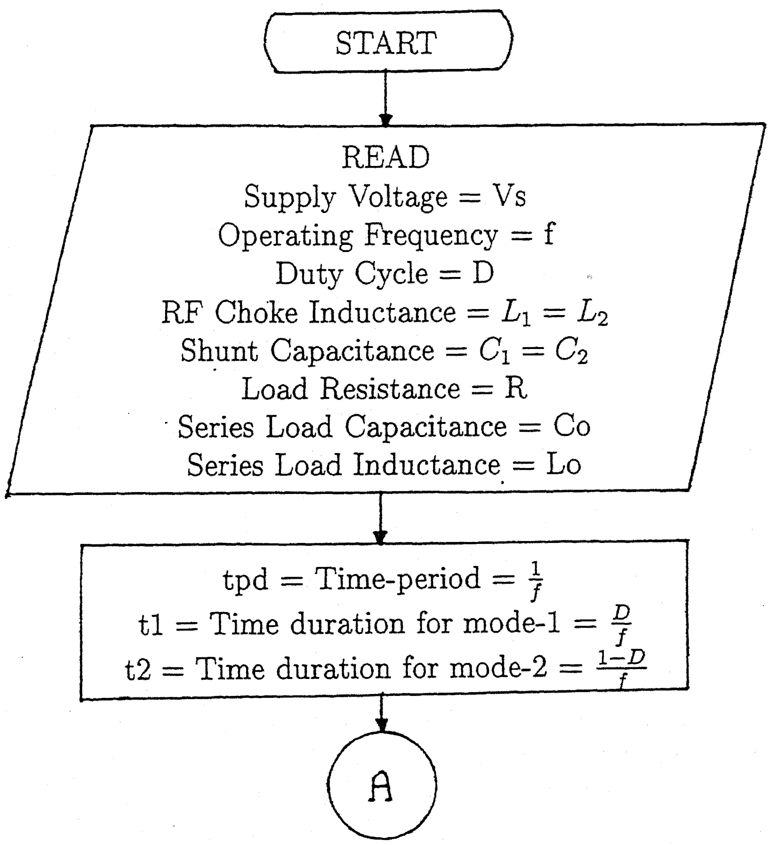
```
      coold=co
      rold=r
      loold=lo
      aac=1.0d0/(2.0d0*pi*w*vs)
      sum1=0.0d0
      sum2=0.0d0
      do 6 i=1,n
      c=dfloat(i)
      sum1=sum1+(1.0d0-(-1.0d0)**i)*iom(i)*dsin(phi(i))/(c**2)
      sum2=sum2+((-1.0d0)**i)*pi*iom(i)*dcos(phi(i))/c
 6    continue
      c1=(0.5d0*pi*pi*i1+sum1-sum2)*aac
      ac=1.0d0/(pi*w*c1)
      do 9 i=1,n
      c=dfloat(i)
      sum1=0.0d0
      sum11=0.0d0
      sum22=0.0d0
      do 8 j=1,n
      e=dfloat(j)
      if(i.eq.j) then
      sum1=sum1+0.5d0*pi*iom(j)*dcos(phi(j))/c
      sum11=sum11-0.5d0*pi*iom(j)*dsin(phi(j))/c
      sum22=sum22+((-1.0)**i)*(1.0-(-1.0)**i)*iom(j)*dcos(phi(j))/(c*e)
      else
      sum2=c**2-e**2
      sum1=sum1+((-1.0d0)**(i+j)-1.0d0)*iom(j)*dsin(phi(j))/sum2
      sum3=e*(c**2-e**2)
      sum11=sum11+c*((-1.0)**(i+j)-1.0)*iom(j)*dcos(phi(j))/sum3
      sum22=sum22+((-1.0)**e)*(1.0-(-1.0)**i)*iom(j)*dcos(phi(j))/(c*e)
      endif
8     continue
      a(i)=ac*(i1*(1.0d0-(-1.0d0)**i)/(c**2)+sum1)
      b(i)=ac*((-pi/c)*i1+sum11+sum22)
9     continue
      zo(1)=sqrt(a(1)**2+b(1)**2)/iom(1)
      sum1=0.0d0
      do 13 i=1,n
      sum1=sum1+iom(i)**2
13    continue
      r=2.0d0*po/sum1
      lo=ql*r/w
      co=1.0d0/(w*w*lo-w*(sqrt(zo(1)*zo(1)-r*r)))
      if((abs(c1-c1old)).gt.0.000001d-6) go to 1100
      if((abs(co-coold)).gt.0.000001d-6)  go to 1100
      if((abs(lo-loold)).gt.0.000001d-5)  go to 1100
      if((abs(r-rold)).gt.0.000001d-6) go to 1100
      write(*,*)'Accurate values of c1,co,r,lo =',c1,co,r,lo
      stop
      end
```

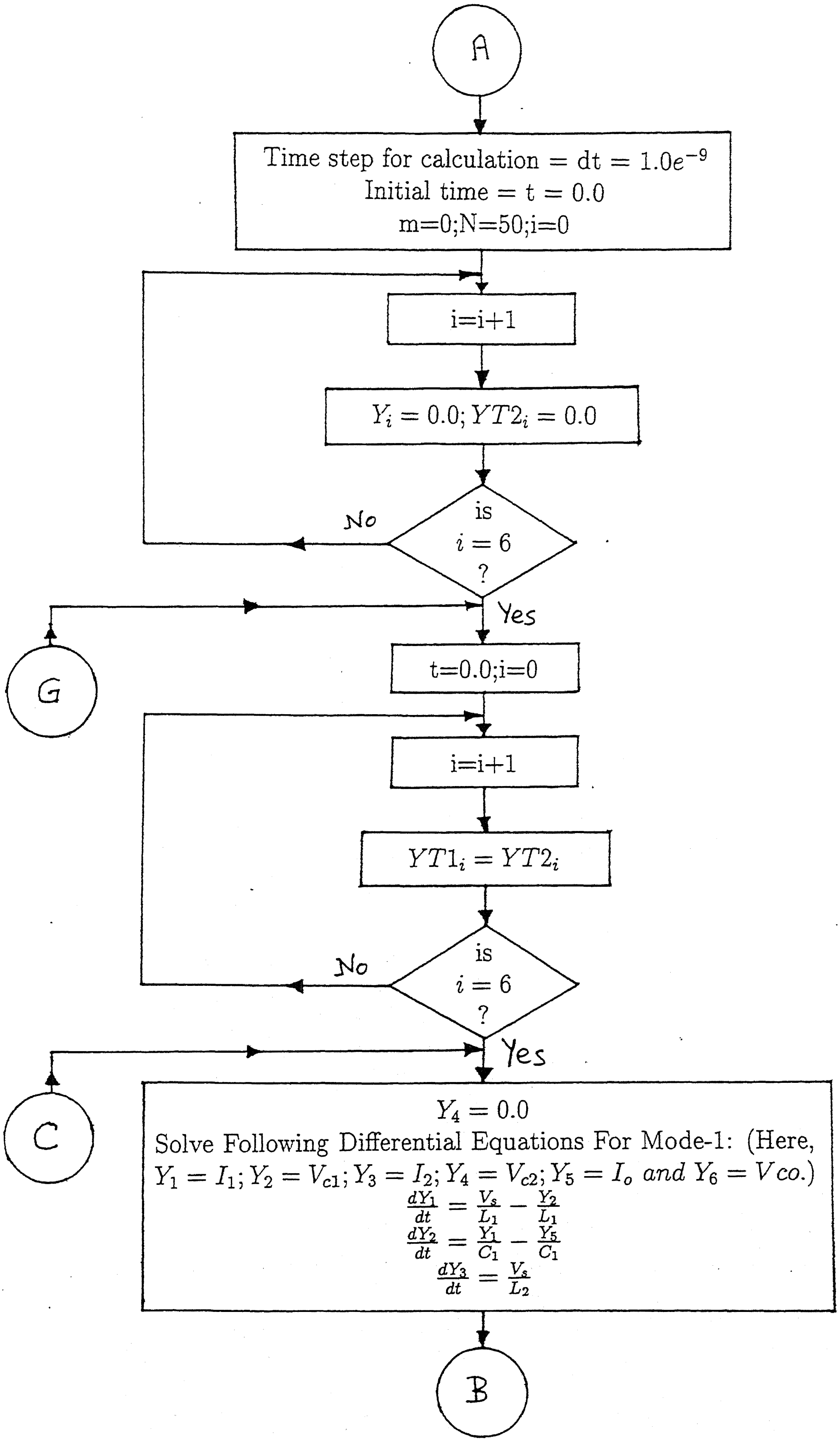
```
C PROGRAM FOR Accurate Design With R, Q1 and f Constant; (Put n>1)
      parameter(n=10)
      double precision i1,po,vs,f,ql,c1,lo,co,pi,r
      double precision w,ac,a(n),b(n),vcn(n),vcnpsi(n)
      double precision zo(n),zopsi(n),c,coold,c1old
      double precision aa1,bb1,aac
      double precision phi(n),iom(n),sum1,sum2,sum3,sum11,sum22
      pi=3.14159270d0
      po=2000.0d0
      ql=40.0d0
      vs=100.0d0
      f=50.00d3
      i1=po/vs
      w=2.0d0*pi*f
      phi(1)=datan(-2.0d0/pi)+pi
      c1=po/(pi*w*(vs**2))
      iom(1)=i1/dsin(phi(1))
      r=po*2.0d0/(iom(1)*iom(1))
      lo=ql*r/w
      ac=1.0d0/(pi*w*c1)
      a(1)=ac*(2.0d0*i1+0.5d0*pi*iom(1)*dcos(phi(1)))
      sum1=2.0d0*iom(1)*dcos(phi(1))
      b(1)=ac*(-pi*i1-0.5d0*pi*iom(1)*dsin(phi(1))-sum1)
      do 1 i=2,n
      c=dfloat(i)
      sum1=(((-1.0d0)**(c+1)-1.0d0)*iom(1)*dsin(phi(1)))/(c**2-1.0d0)
      a(i)=ac*(i1*(1.0d0-(-1.0d0)**c)/(c**2)+sum1)
 1    continue
      do 2 i=2,n
      c=dfloat(i)
      sum1=-(1.0d0-(-1)**c)*iom(1)*dcos(phi(1))/c
      sum2=c*(-1.0d0+(-1.0)**(c+1.0))*iom(1)*dcos(phi(1))/(c**2-1.0)
      b(i)=ac*(-(pi/c)*i1+sum2+sum1)
 2    continue
      zo(1)=sqrt(a(1)**2+b(1)**2)/(iom(1))
      co=1.0d0/(w*w*lo-w*(sqrt(zo(1)*zo(1)-r*r)))
      write(*,*)'Approximate Values of c1,co,r,lo=',c1,co,r,lo
1100  do 3 i=1,n
      c=dfloat(i)
      vcn(i)=sqrt(a(i)**2+b(i)**2)
      zo(i)=sqrt(r**2+(c*w*lo-(1.0d0/(c*w*co)))**2)
      zopsi(i)=atan((c*w*lo-(1.0d0/(c*w*co)))/r)
      iom(i)=vcn(i)/zo(i)
      if(b(i).lt.0.0d0)then
      vcnpsi(i)=atan(a(i)/b(i))+pi
      else
      vcnpsi(i)=atan(a(i)/b(i))
      endif
      phi(i)=vcnpsi(i)-zopsi(i)
 3    continue
      sum1=0.0d0
      sum2=0.0d0
      do 4 i=2,n
      c=dfloat(i)
      sum1=sum1+(1.0d0-(-1.0d0)**i)*iom(i)*dcos(phi(i))/c
      sum2=sum2+iom(i)*dsin(phi(i))
 4    continue
      aa1=-0.5d0*(pi*i1+sum1)
      bb1=i1-sum2
      iom(1)=sqrt(aa1**2+bb1**2)
      if(aa1.lt.0.0d0)then
      phi(1)=atan(bb1/aa1)+pi
      else
      phi(1)=atan(bb1/aa1)
      endif
      c1old=c1
```

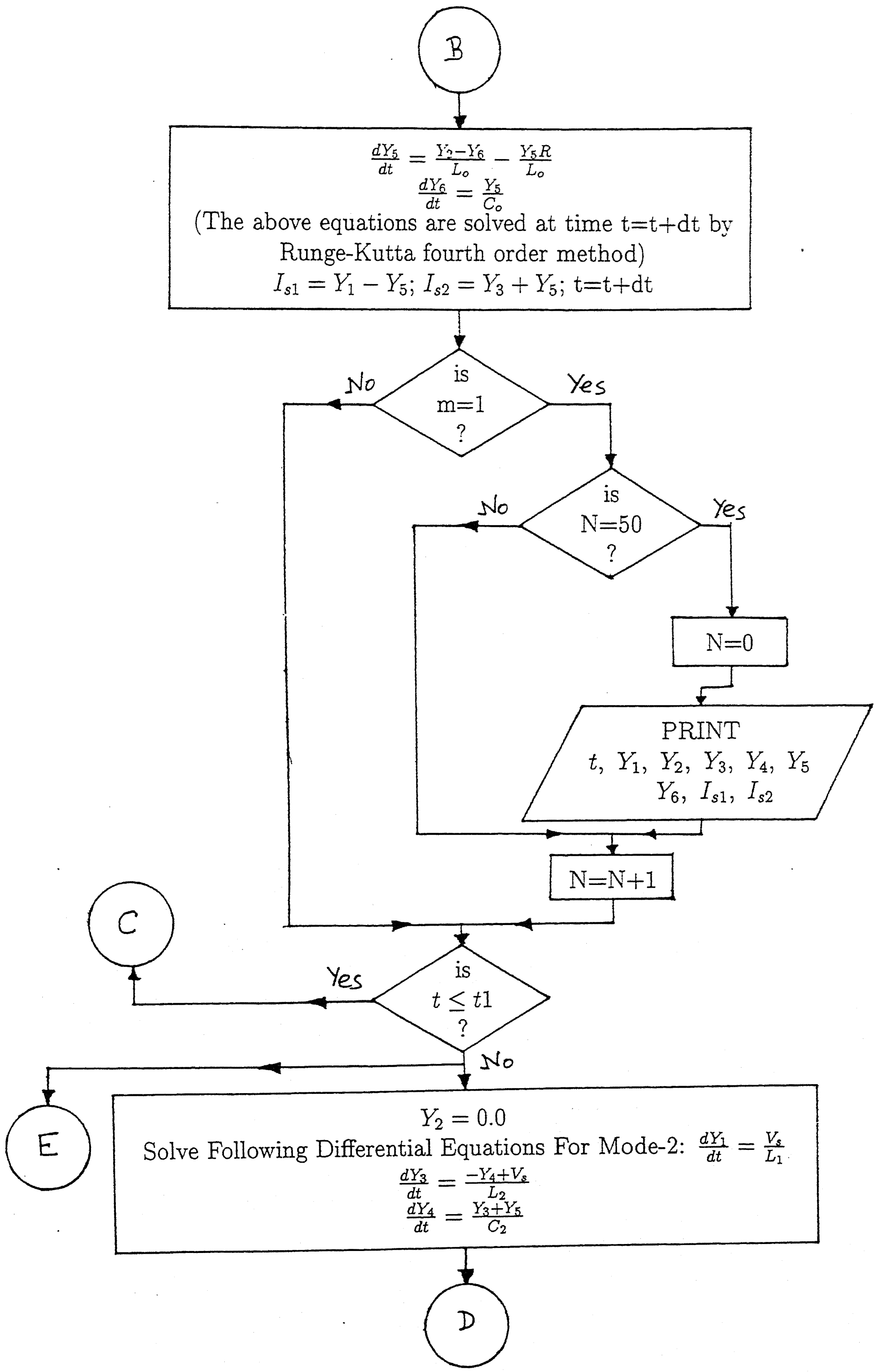
```
      coold=co
      aac=1.0d0/(2.0d0*pi*w*vs)
      sum1=0.0d0
      sum2=0.0d0
      do 6 i=1,n
      c=dfloat(i)
      sum1=sum1+(1.0d0-(-1.0d0)**i)*iom(i)*dsin(phi(i))/(c**2)
      sum2=sum2+((-1.0d0)**i)*pi*iom(i)*dcos(phi(i))/c
 6    continue
      c1=(0.5d0*pi*pi*i1+sum1-sum2)*aac
      ac=1.0d0/(pi*w*c1)
      do 9 i=1,n
      c=dfloat(i)
      sum1=0.0d0
      sum11=0.0d0
      sum22=0.0d0
      do 8 j=1,n
      e=dfloat(j)
      if(i.eq.j) then
      sum1=sum1+0.5d0*pi*iom(j)*dcos(phi(j))/c
      sum11=sum11-0.5d0*pi*iom(j)*dsin(phi(j))/c
      sum22=sum22+((-1.0)**i)*(1.0-(-1.0)**i)*iom(j)*dcos(phi(j))/(c*e)
      else
      sum2=c**2-e**2
      sum1=sum1+((-1.0d0)**(i+j)-1.0d0)*iom(j)*dsin(phi(j))/sum2
      sum3=e*(c**2-e**2)
      sum11=sum11+c*((-1.0)**(i+j)-1.0)*iom(j)*dcos(phi(j))/sum3
      sum22=sum22+((-1.0)**e)*(1.0-(-1.0)**i)*iom(j)*dcos(phi(j))/(c*e)
      endif
 8    continue
      a(i)=ac*(i1*(1.0d0-(-1.0d0)**i)/(c**2)+sum1)
      b(i)=ac*((-pi/c)*i1+sum11+sum22)
 9    continue
      zo(1)=sqrt(a(1)**2+b(1)**2)/iom(1)
      co=1.0d0/(w*w*lo-w*(sqrt(zo(1)*zo(1)-r*r)))
      sum1=0.0d0
      do 13 i=1,n
      sum1=sum1+iom(i)**2
 13   continue
      po=r*sum1/2.0d0
      vs=po/i1
      if((abs(c1-c1old)).gt.0.0000001d-6) go to 1100
      if((abs(co-coold)).gt.0.0000001d-6)  go to 1100
      write(*,*)'Accurate values of c1,co,r,lo,vs =',c1,co,r,lo
      stop
      end
```

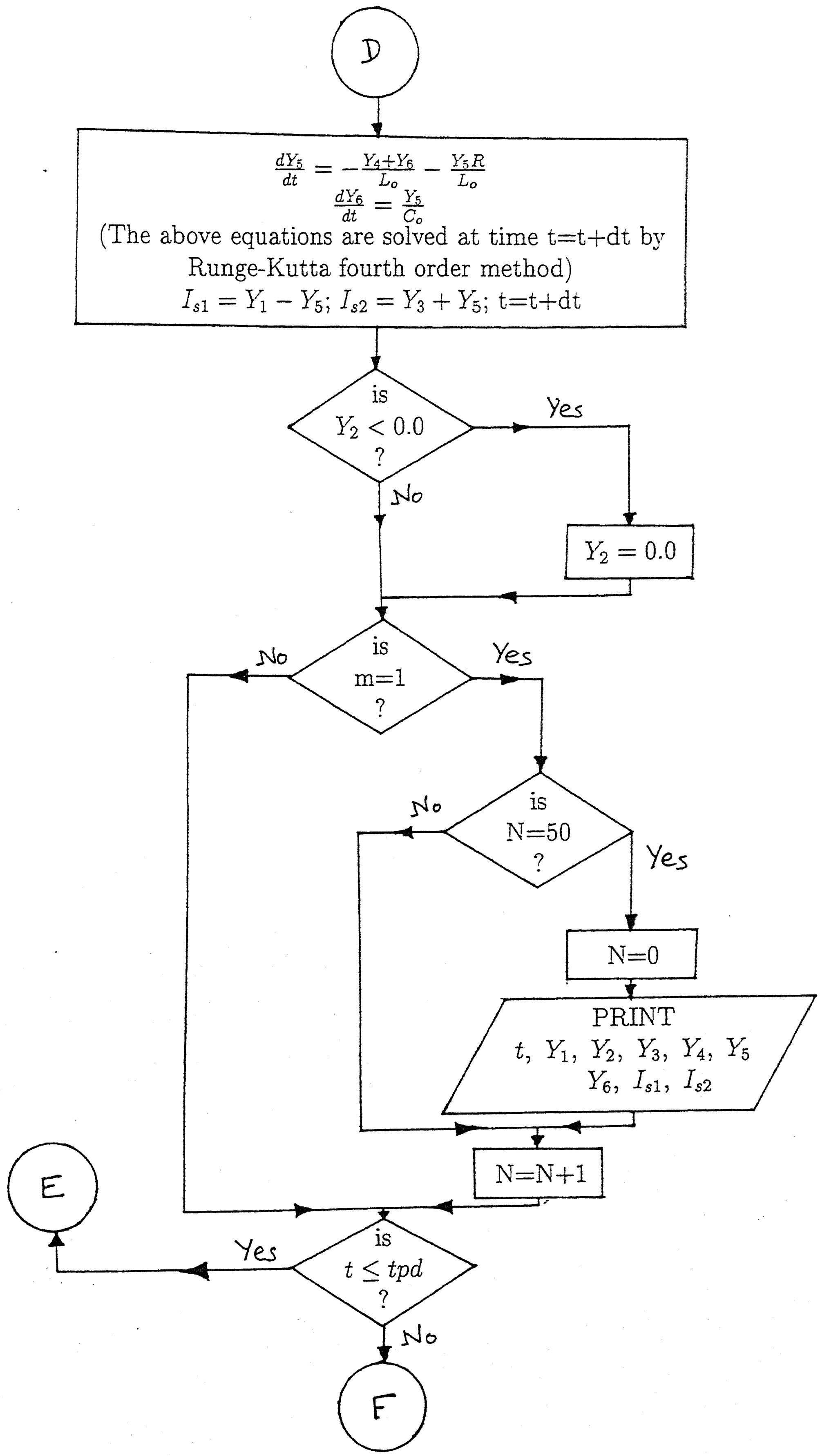
Appendix D

Flow-Chart For Exact Analysis of Class-E Push-Pull Amplifier With Two Modes of Operation (Using State-Equations)

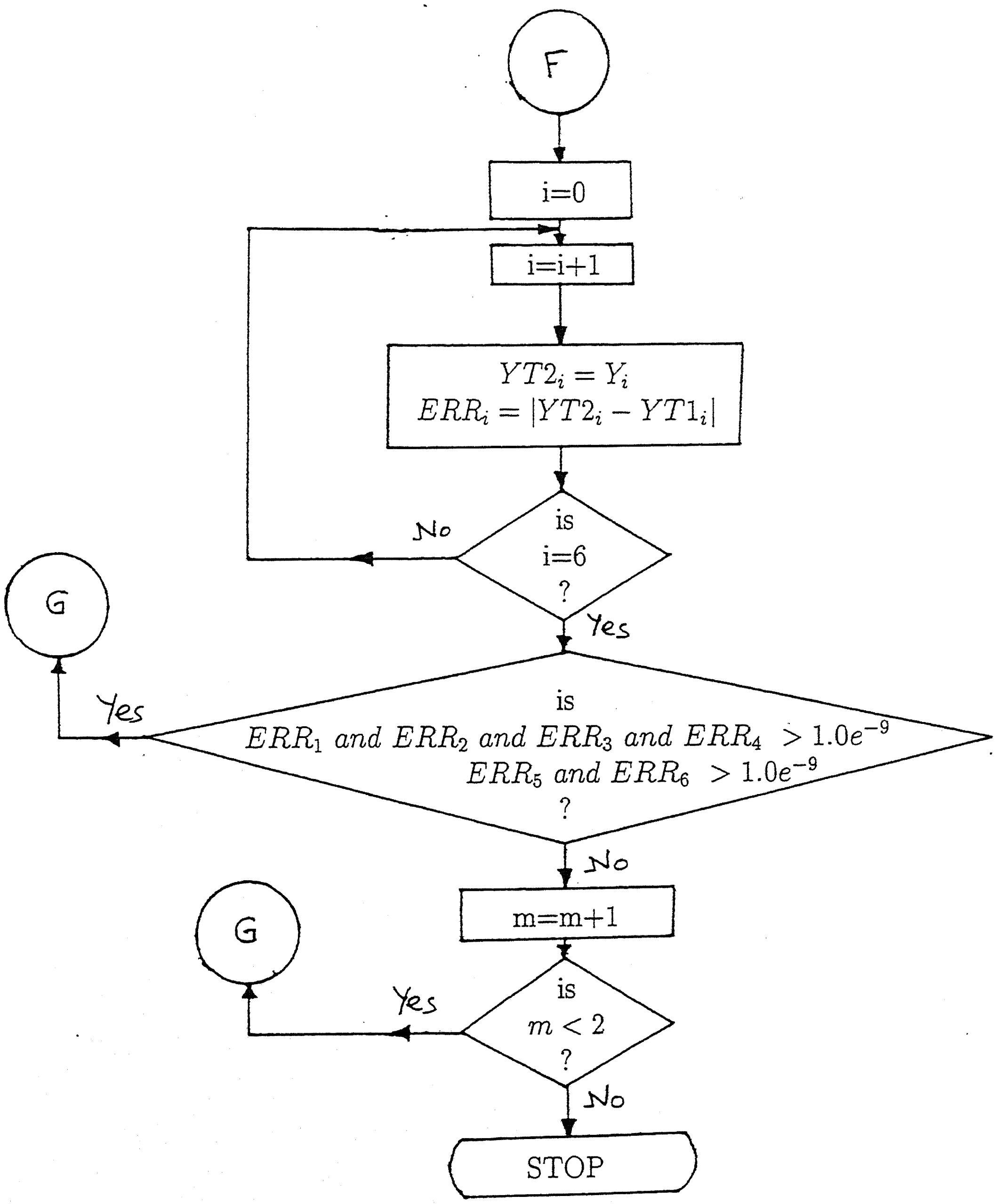



A
Time step for calculation = dt = $1.0e^{-9}$
Initial time = t = 0.0
m=0;N=50;i=0
i=i+1
$Y_i = 0.0; YT2_i = 0.0$
is $i = 6$?
No
Yes
G
t=0.0;i=0
i=i+1
$YT1_i = YT2_i$
is $i = 6$?
No
Yes
C
$Y_4 = 0.0$
Solve Following Differential Equations For Mode-1: (Here, $Y_1 = I_1; Y_2 = V_{c1}; Y_3 = I_2; Y_4 = V_{c2}; Y_5 = I_o$ and $Y_6 = Vco.$)
$\frac{dY_1}{dt} = \frac{V_s}{L_1} - \frac{Y_2}{L_1}$
$\frac{dY_2}{dt} = \frac{Y_1}{C_1} - \frac{Y_5}{C_1}$
$\frac{dY_3}{dt} = \frac{V_s}{L_2}$
B


B
$\frac{dY_5}{dt} = \frac{Y_2 - Y_6}{L_o} - \frac{Y_5 R}{L_o}$
$\frac{dY_6}{dt} = \frac{Y_5}{C_o}$
(The above equations are solved at time t=t+dt by Runge-Kutta fourth order method)
$I_{s1} = Y_1 - Y_5$; $I_{s2} = Y_3 + Y_5$; t=t+dt
is m=1 ?
No
Yes
is N=50 ?
No
Yes
N=0
PRINT
$t, Y_1, Y_2, Y_3, Y_4, Y_5$
Y_6, I_{s1}, I_{s2}
N=N+1
C
is $t \leq t1$?
Yes
No
E
$Y_2 = 0.0$
Solve Following Differential Equations For Mode-2: $\frac{dY_1}{dt} = \frac{V_s}{L_1}$
$\frac{dY_3}{dt} = \frac{-Y_4 + V_s}{L_2}$
$\frac{dY_4}{dt} = \frac{Y_3 + Y_5}{C_2}$
D



D
$\frac{dY_5}{dt} = -\frac{Y_4+Y_6}{L_o} - \frac{Y_5 R}{L_o}$
$\frac{dY_6}{dt} = \frac{Y_5}{C_o}$
(The above equations are solved at time t=t+dt by Runge-Kutta fourth order method)
$I_{s1} = Y_1 - Y_5$; $I_{s2} = Y_3 + Y_5$; t=t+dt
is $Y_2 < 0.0$?
Yes
No
$Y_2 = 0.0$
is m=1 ?
No
Yes
is N=50 ?
No
Yes
N=0
PRINT
t, Y_1, Y_2, Y_3, Y_4, Y_5
Y_6, I_{s1}, I_{s2}
N=N+1
is $t \leq tpd$?
Yes
No
E
F


F
i=0
i=i+1
$YT2_i = Y_i$
$ERR_i = |YT2_i - YT1_i|$
is
i=6
?
No
Yes
G
is
ERR_1 and ERR_2 and ERR_3 and ERR_4 $> 1.0e^{-9}$
ERR_5 and ERR_6 $> 1.0e^{-9}$
?
Yes
No
m=m+1
G
is
$m < 2$
?
Yes
No
STOP

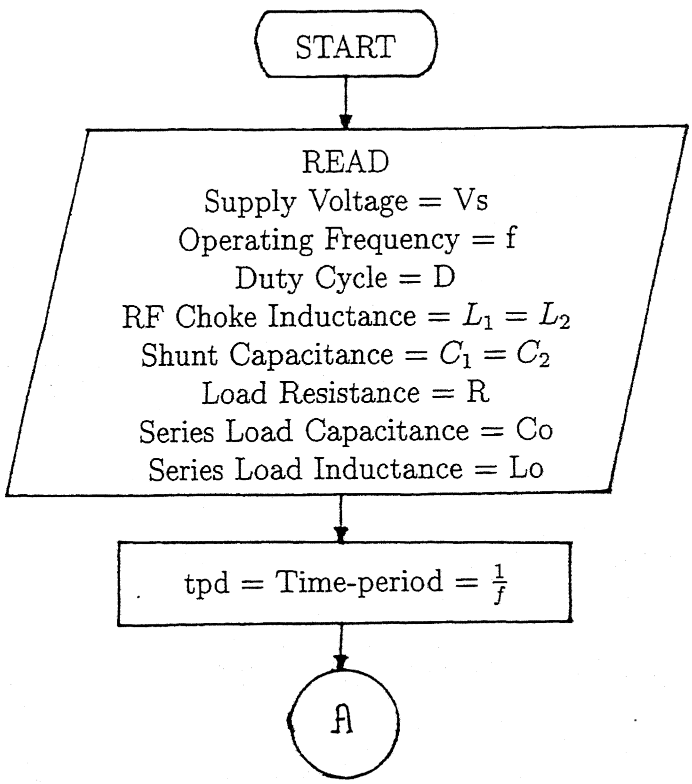
```
C PROGRAM FOR PUSH-PULL CLASS-E AMPLIFIER (WITH D=0.5).
        double precision y(6),yy(6),yyy(6),yyyy(6)
        double precision rka(6),rkb(6),rkc(6),rkd(6)
        double precision vs,r,l1,l2,c1,c2,lo,co,t,f,tpd
        double precision t1,t2,ss1,ss2,d
        double precision yt1(6),yt2(6),err(6)
        open(unit=21,file='tt.i1')
        open(unit=22,file='tt.vc1')
        open(unit=23;file='tt.i2')
        open(unit=24,file='tt.vc2')
        open(unit=25,file='tt.io')
        open(unit=26,file='tt.vco')
        open(unit=27,file='tt.is1')
        open(unit=28,file='tt.is2')
        vs=100.0d0
        r=5.768d0
        l1=5.0d-3
        l2=5.0d-3
        c1=0.20264d-6
        c2=0.20264d-6
        lo=183.601d-6
        co=0.062374d-6
        f=50.0d3
        tpd=1.0d0/f
        d=0.50d0
        t1=tpd*d
        t2=tpd*(1.0d0-d)
        do 1 i=1,6
        y(i)=0.0d0
        yt2(i)=0.0d0
  1     continue
        dt=1.0d-9
        m=0
        nn=50
 1000   t=0.0d0
        do 10 i=1,6
        yt1(i)=yt2(i)
 10     continue
c FOR CASE OF  S1 _ OFF & S2 - ON
c---------------------------------
 100    y(4)=0.0d0
        rka(1)=dt*(vs/l1-y(2)/l1)
        rka(2)=dt*(y(1)/c1-y(5)/c1)
        rka(3)=dt*(vs/l2)
        rka(5)=dt*((y(2)-y(6))/lo-y(5)*r/lo)
        rka(6)=dt*y(5)/co
        do 5 i=1,6
  5     yy(i)=y(i)+rka(i)/2.0d0
        rkb(1)=dt*(vs/l1-yy(2)/l1)
        rkb(2)=dt*(yy(1)/c1-yy(5)/c1)
        rkb(3)=dt*(vs/l2)
        rkb(5)=dt*((yy(2)-yy(6))/lo-yy(5)*r/lo)
        rkb(6)=dt*yy(5)/co
        do 6 i=1,6
  6     yyy(i)=yy(i)+rkb(i)/2.0d0
        rkc(1)=dt*(vs/l1-yyy(2)/l1)
        rkc(2)=dt*(yyy(1)/c1-yyy(5)/c1)
        rkc(3)=dt*(vs/l2)
        rkc(5)=dt*((yyy(2)-yyy(6))/lo-yyy(5)*r/lo)
        rkc(6)=dt*yyy(5)/co
        do 7 i=1,6
  7     yyyy(i)=yyy(i)+rkc(i)
        rkd(1)=dt*(vs/l1-yyyy(2)/l1)
        rkd(2)=dt*(yyyy(1)/c1-yyyy(5)/c1)
        rkd(3)=dt*(vs/l2)
```

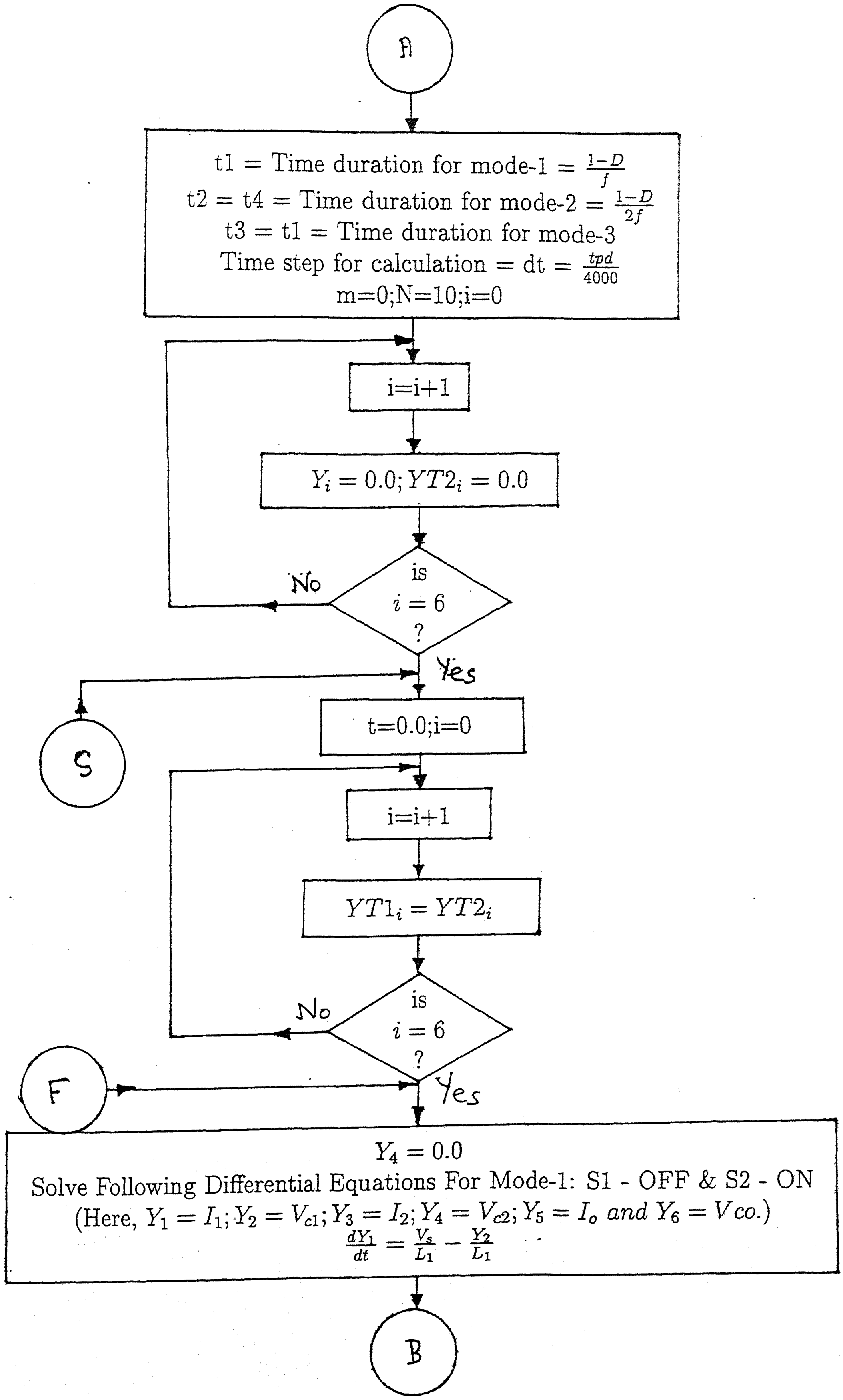
```
      rkd(5)=dt*((yyyy(2)-yyyy(6))/lo-yyyy(5)*r/lo)
      rkd(6)=dt*yyyy(5)/co
      do 8 i=1,6
 8    y(i)=y(i)+(1.0d0/6.0d0)*(rka(i)+2.0d0*rkb(i)+2.0d0*rkc(i)+rkd(i
      t=t+dt
      if(y(2).le.0.0d0)then
      y(2)=0.0d0
      endif
      ss1=y(1)-y(5)
      ss2=y(3)+y(5)
      y(4)=0.0d0
      if(m.ne.1) go to 3
      if(nn.eq.50) then
      write(21,*)t,y(1)
      write(22,*)t,y(2)
      write(23,*)t,y(3)
      write(24,*)t,y(4)
      write(25,*)t,y(5)
      write(26,*)t,y(6)
      write(27,*)t,ss1
      write(28,*)t,ss2
      nn=0
      endif
      nn=nn+1
 3    if(t.lt.t1) go to 100
c FOR CASE OF  S1 _ ON & S2 - OFF
c--------------------------------
 300  rka(1)=dt*(vs/l1)
      rka(3)=dt*(-y(4)/l2+vs/l2)
      rka(4)=dt*(y(3)+y(5))/c2
      rka(5)=-dt*((y(4)+y(6))/lo+y(5)*r/lo)
      rka(6)=dt*y(5)/co
      do 9 i=1,6
 9    yy(i)=y(i)+rka(i)/2.0d0
      rkb(1)=dt*(vs/l1)
      rkb(3)=dt*(-yy(4)/l2+vs/l2)
      rkb(4)=dt*(yy(3)+yy(5))/c2
      rkb(5)=-dt*((yy(4)+yy(6))/lo+yy(5)*r/lo)
      rkb(6)=dt*yy(5)/co
      do 66 i=1,6
 66   yyy(i)=yy(i)+rkb(i)/2.0d0
      rkc(1)=dt*(vs/l1)
      rkc(3)=dt*(-yyy(4)/l2+vs/l2)
      rkc(4)=dt*(yyy(3)+yyy(5))/c2
      rkc(5)=-dt*((yyy(4)+yyy(6))/lo+yyy(5)*r/lo)
      rkc(6)=dt*yyy(5)/co
      do 77 i=1,6
 77   yyyy(i)=yyy(i)+rkc(i)
      rkd(1)=dt*(vs/l1)
      rkd(3)=dt*(-yyyy(4)/l2+vs/l2)
      rkd(4)=dt*(yyyy(3)+yyyy(5))/c2
      rkd(5)=-dt*((yyyy(4)+yyyy(6))/lo+yyyy(5)*r/lo)
      rkd(6)=dt*yyyy(5)/co
      do 88 i=1,6
 88   y(i)=y(i)+(1.0d0/6.0d0)*(rka(i)+2.0d0*rkb(i)+2.0d0*rkc(i)+rkd(i)
      y(2)=0.0d0
      t=t+dt
      if(y(4).le.0.0d0)then
      y(4)=0.0d0
      endif
      ss1=y(1)-y(5)
      ss2=y(3)+y(5)
      if(m.ne.1) go to 33
      if(nn.eq.50) then
      write(21,*)t,y(1)
      write(22,*)t,y(2)
```

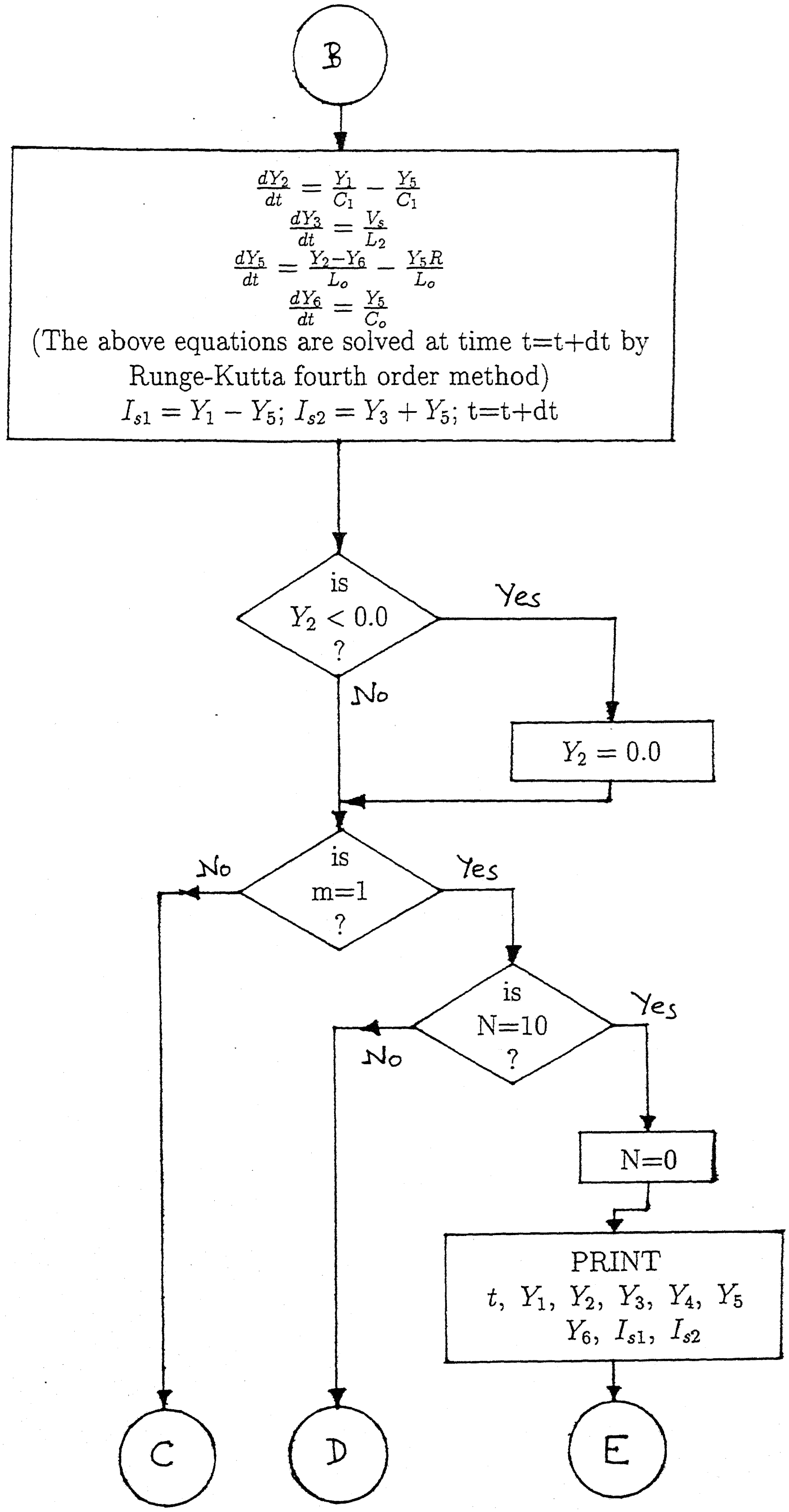
```
      write(23,*)t,y(3)
      write(24,*)t,y(4)
      write(25,*)t,y(5)
      write(26,*)t,y(6)
      write(27,*)t,ss1
      write(28,*)t,ss2
      nn=0
      endif
      nn=nn+1
33    if(t.le.(t1+t2)) go to 300
      do 12 i=1,6
      yt2(i)=y(i)
      err(i)=abs(yt2(i)-yt1(i))
12    continue
      if(err(1) .gt. 0.1d-8) go to 1000
      if(err(2) .gt. 0.1d-8) go to 1000
      if(err(3) .gt. 0.1d-8) go to 1000
      if(err(4) .gt. 0.1d-8) go to 1000
      if(err(5) .gt. 0.1d-8) go to 1000
      if(err(6) .gt. 0.1d-8) go to 1000
      m=m+1
      if(m.lt.2) go to 1000
      close(21)
      close(22)
      close(23)
      close(24)
      close(25)
      close(26)
      close(27)
      close(28)
      stop
      end
```

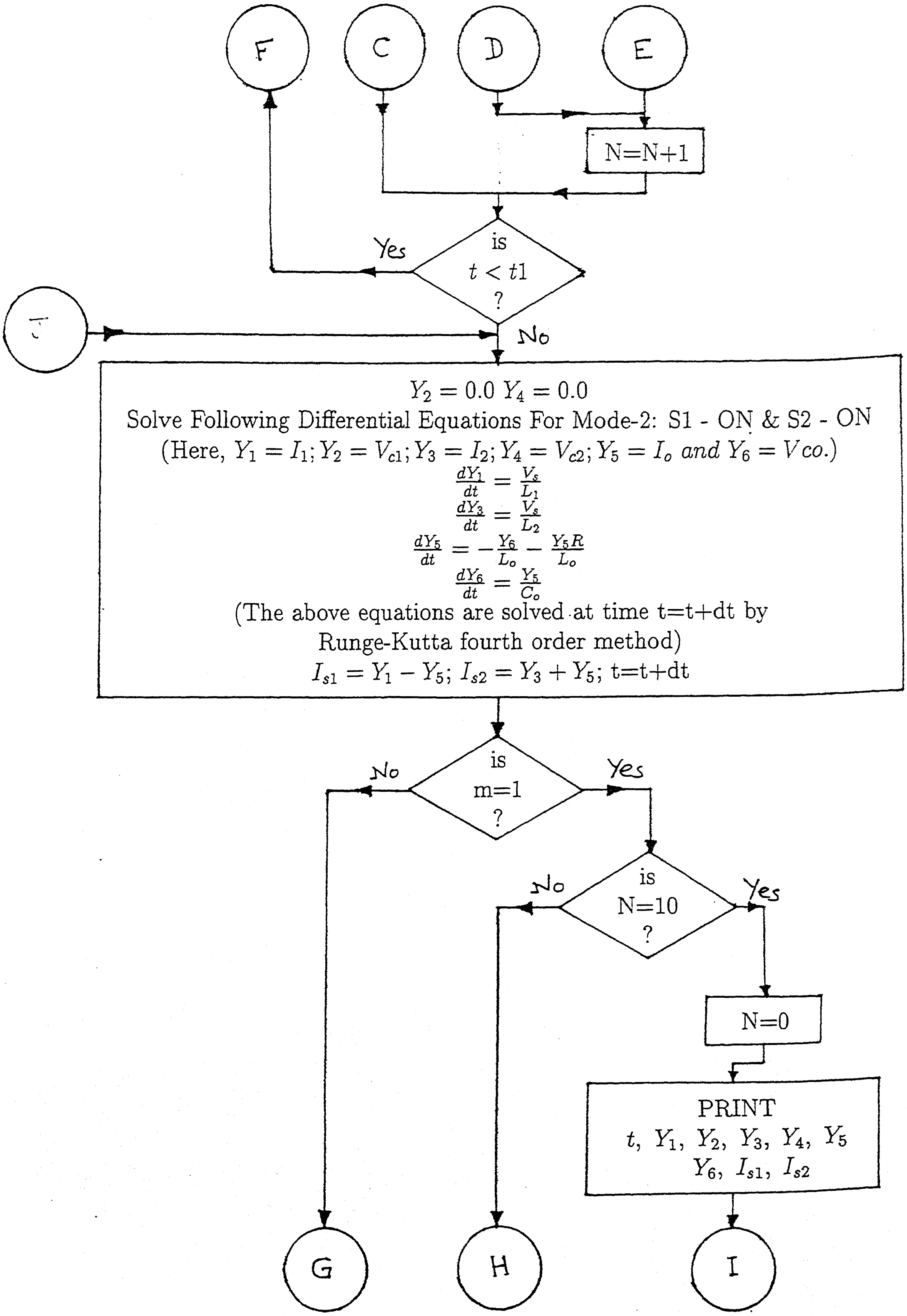
Appendix E

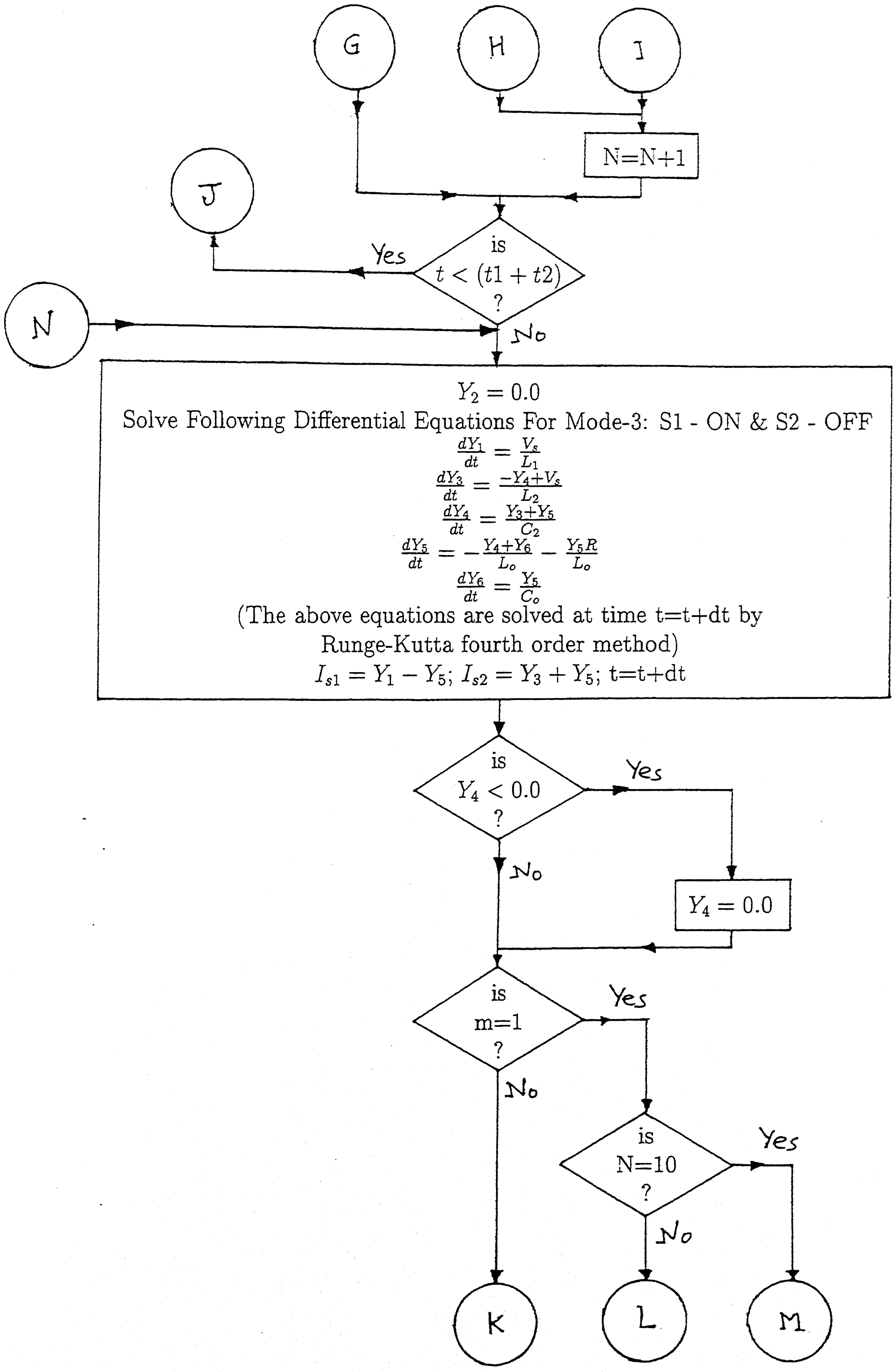
Flow-Chart For Exact Analysis of Class-E Push-Pull Amplifier With Three Modes of Operation (Using State-Equations)

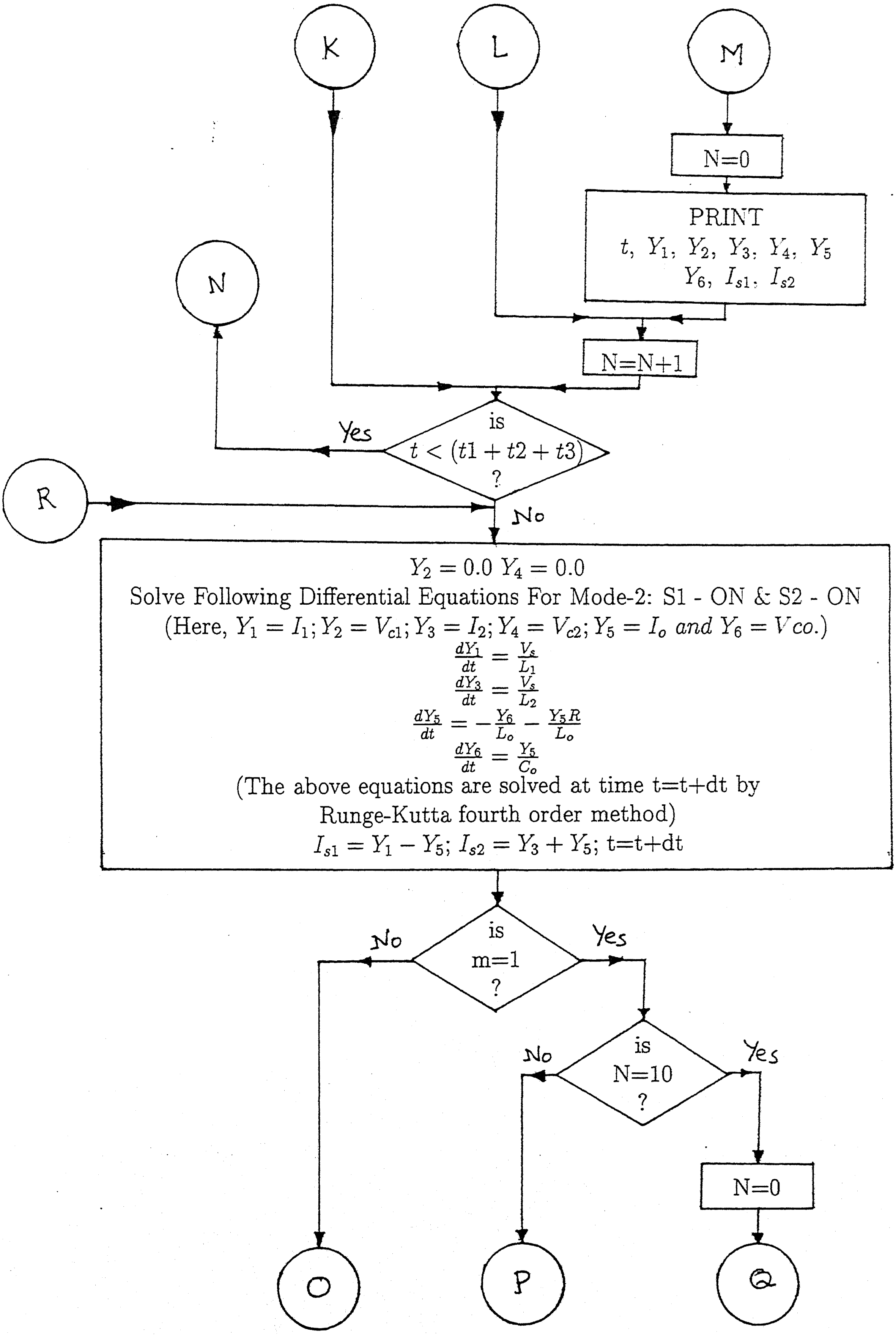


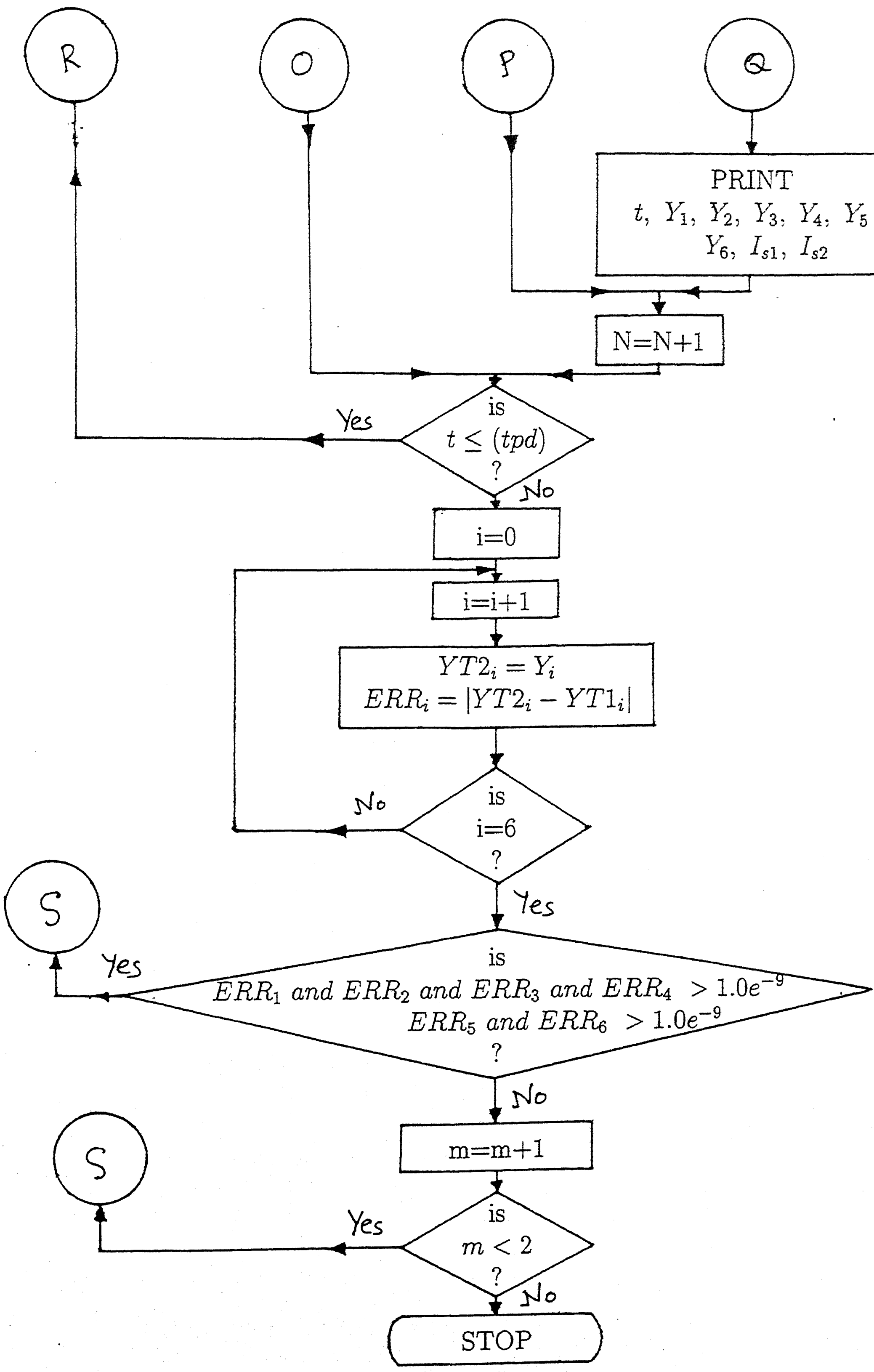

A
t1 = Time duration for mode-1 = $\frac{1-D}{f}$
t2 = t4 = Time duration for mode-2 = $\frac{1-D}{2f}$
t3 = t1 = Time duration for mode-3
Time step for calculation = dt = $\frac{tpd}{4000}$
m=0;N=10;i=0
i=i+1
$Y_i = 0.0; YT2_i = 0.0$
is $i = 6$?
No
Yes
S
t=0.0;i=0
i=i+1
$YT1_i = YT2_i$
is $i = 6$?
No
Yes
F
$Y_4 = 0.0$
Solve Following Differential Equations For Mode-1: S1 - OFF & S2 - ON
(Here, $Y_1 = I_1; Y_2 = V_{c1}; Y_3 = I_2; Y_4 = V_{c2}; Y_5 = I_o$ and $Y_6 = Vco$.)
$\frac{dY_1}{dt} = \frac{V_s}{L_1} - \frac{Y_2}{L_1}$
B


B
$\frac{dY_2}{dt} = \frac{Y_1}{C_1} - \frac{Y_5}{C_1}$
$\frac{dY_3}{dt} = \frac{V_s}{L_2}$
$\frac{dY_5}{dt} = \frac{Y_2 - Y_6}{L_o} - \frac{Y_5 R}{L_o}$
$\frac{dY_6}{dt} = \frac{Y_5}{C_o}$
(The above equations are solved at time t=t+dt by Runge-Kutta fourth order method)
$I_{s1} = Y_1 - Y_5$; $I_{s2} = Y_3 + Y_5$; t=t+dt
is $Y_2 < 0.0$?
Yes
No
$Y_2 = 0.0$
is m=1 ?
No
Yes
is N=10 ?
No
Yes
N=0
PRINT
$t,\ Y_1,\ Y_2,\ Y_3,\ Y_4,\ Y_5$
$Y_6,\ I_{s1},\ I_{s2}$
C
D
E


F
C
D
E
N=N+1
Yes
is $t < t1$?
J
No
$Y_2 = 0.0$ $Y_4 = 0.0$
Solve Following Differential Equations For Mode-2: S1 - ON & S2 - ON
(Here, $Y_1 = I_1; Y_2 = V_{c1}; Y_3 = I_2; Y_4 = V_{c2}; Y_5 = I_o$ and $Y_6 = Vco$.)
$\frac{dY_1}{dt} = \frac{V_s}{L_1}$
$\frac{dY_3}{dt} = \frac{V_s}{L_2}$
$\frac{dY_5}{dt} = -\frac{Y_6}{L_o} - \frac{Y_5 R}{L_o}$
$\frac{dY_6}{dt} = \frac{Y_5}{C_o}$
(The above equations are solved at time t=t+dt by
Runge-Kutta fourth order method)
$I_{s1} = Y_1 - Y_5$; $I_{s2} = Y_3 + Y_5$; t=t+dt
No
is m=1 ?
Yes
No
is N=10 ?
Yes
N=0
PRINT
t, Y_1, Y_2, Y_3, Y_4, Y_5
Y_6, I_{s1}, I_{s2}
G
H
I


G
H
I
N=N+1
J
Yes
is $t < (t1 + t2)$?
N
No
$Y_2 = 0.0$
Solve Following Differential Equations For Mode-3: S1 - ON & S2 - OFF
$\frac{dY_1}{dt} = \frac{V_s}{L_1}$
$\frac{dY_3}{dt} = \frac{-Y_4+V_s}{L_2}$
$\frac{dY_4}{dt} = \frac{Y_3+Y_5}{C_2}$
$\frac{dY_5}{dt} = -\frac{Y_4+Y_6}{L_o} - \frac{Y_5 R}{L_o}$
$\frac{dY_6}{dt} = \frac{Y_5}{C_o}$
(The above equations are solved at time t=t+dt by
Runge-Kutta fourth order method)
$I_{s1} = Y_1 - Y_5$; $I_{s2} = Y_3 + Y_5$; t=t+dt
is $Y_4 < 0.0$?
Yes
No
$Y_4 = 0.0$
is m=1 ?
Yes
No
is N=10 ?
Yes
No
K
L
M


K
L
M
N=0
PRINT
$t,\ Y_1,\ Y_2,\ Y_3,\ Y_4,\ Y_5$
$Y_6,\ I_{s1},\ I_{s2}$
N
N=N+1
Yes
is
$t < (t1 + t2 + t3)$
?
R
No
$Y_2 = 0.0\ Y_4 = 0.0$
Solve Following Differential Equations For Mode-2: S1 - ON & S2 - ON
(Here, $Y_1 = I_1; Y_2 = V_{c1}; Y_3 = I_2; Y_4 = V_{c2}; Y_5 = I_o$ and $Y_6 = Vco.$)
$\frac{dY_1}{dt} = \frac{V_s}{L_1}$
$\frac{dY_3}{dt} = \frac{V_s}{L_2}$
$\frac{dY_5}{dt} = -\frac{Y_6}{L_o} - \frac{Y_5 R}{L_o}$
$\frac{dY_6}{dt} = \frac{Y_5}{C_o}$
(The above equations are solved at time t=t+dt by
Runge-Kutta fourth order method)
$I_{s1} = Y_1 - Y_5;\ I_{s2} = Y_3 + Y_5;$ t=t+dt
No
is
m=1
?
Yes
No
is
N=10
?
Yes
N=0
O
P
Q


R
O
P
Q
PRINT
$t,\ Y_1,\ Y_2,\ Y_3,\ Y_4,\ Y_5$
$Y_6,\ I_{s1},\ I_{s2}$
N=N+1
is
$t \leq (tpd)$
?
Yes
No
i=0
i=i+1
$YT2_i = Y_i$
$ERR_i = |YT2_i - YT1_i|$
is
i=6
?
No
Yes
S
is
ERR_1 and ERR_2 and ERR_3 and ERR_4 $> 1.0e^{-9}$
ERR_5 and ERR_6 $> 1.0e^{-9}$
?
Yes
No
m=m+1
S
is
$m < 2$
?
Yes
No
STOP

```
C PROGRAM FOR CLASS-E TUNED AMPLIFIER WITH 3-MODE OPERATION (D=2/3).
        double precision y(6),yy(6),yyy(6),yyyy(6)
        double precision rka(6),rkb(6),rkc(6),rkd(6)
        double precision vs,r,l1,l2,c1,c2,lo,co,t,f,tpd,d
        double precision t1,t2,t3,t4,ss1,ss2
        double precision yt1(6),yt2(6),err(6)
        open(unit=21,file='tn.i1')
        open(unit=22,file='tn.vc1')
        open(unit=23,file='tn.i2')
        open(unit=24,file='tn.vc2')
        open(unit=25,file='tn.io')
        open(unit=26,file='tn.vco')
        open(unit=27,file='tn.is1')
        open(unit=28,file='tn.is2')
        vs=100.0d0
        f=50.0d3
        d=2.0d0/3.0d0
        l1=5.0d-3
        l2=l1
        c1=0.03168d-6
        c2=0.03168d-6
        r=11.9716d0
        co=0.02839d-6
        lo=381.0682d-6
        tpd=1.0d0/f
        t1=tpd*(1.0d0-d)
        t3=t1
        t2=tpd/6.0d0
        t2=tpd*(1.0d0-d)+0.5d0
        t4=t2
        dt=tpd/4000.0d0
        m=0
        n=10
        do 1 i=1,6
        y(i)=0.0d0
        yt2(i)=0.0d0
1       continue
1000    t=0.0d0
        do 10 i=1,6
        yt1(i)=yt2(i)
10      continue
c FOR CASE OF  S1 - OFF & S2 - ON
c--------------------------------
100     y(4)=0.0d0
        rka(1)=dt*(vs/l1-y(2)/l1)
        rka(2)=dt*(y(1)/c1-y(5)/c1)
        rka(3)=dt*(vs/l2)
        rka(5)=dt*((y(2)-y(6))/lo-y(5)*r/lo)
        rka(6)=dt*y(5)/co
        do 5 i=1,6
5       yy(i)=y(i)+rka(i)/2.0d0
        rkb(1)=dt*(vs/l1-yy(2)/l1)
        rkb(2)=dt*(yy(1)/c1-yy(5)/c1)
        rkb(3)=dt*(vs/l2)
        rkb(5)=dt*((yy(2)-yy(6))/lo-yy(5)*r/lo)
        rkb(6)=dt*yy(5)/co
        do 6 i=1,6
6       yyy(i)=yy(i)+rkb(i)/2.0d0
        rkc(1)=dt*(vs/l1-yyy(2)/l1)
        rkc(2)=dt*(yyy(1)/c1-yyy(5)/c1)
        rkc(3)=dt*(vs/l2)
        rkc(5)=dt*((yyy(2)-yyy(6))/lo-yyy(5)*r/lo)
        rkc(6)=dt*yyy(5)/co
        do 7 i=1,6
```

```
 7      yyyy(i)=yyy(i)+rkc(i)
        rkd(1)=dt*(vs/l1-yyyy(2)/l1)
        rkd(2)=dt*(yyyy(1)/c1-yyyy(5)/c1)
        rkd(3)=dt*(vs/l2)
        rkd(5)=dt*((yyyy(2)-yyyy(6))/lo-yyyy(5)*r/lo)
        rkd(6)=dt*yyyy(5)/co
        do 8 i=1,6
 8      y(i)=y(i)+(1.0d0/6.0d0)*(rka(i)+2.0d0*rkb(i)+2.0d0*rkc(i)+rkd(i)
        t=t+dt
        if(y(2).lt.0.0d0)then
        y(2)=0.0d0
        endif
        ss1=y(1)-y(5)
        ss2=y(3)+y(5)
        y(4)=0.0d0
        if(m.ne.1) go to 3
        if(n.eq.10) then
        n=0
        write(21,*)t,y(1)
        write(22,*)t,y(2)
        write(23,*)t,y(3)
        write(24,*)t,y(4)
        write(25,*)t,y(5)
        write(26,*)t,y(6)
        write(27,*)t,ss1
        write(28,*)t,ss2
        endif
        n=n+1
 3      if(t.lt.t1) go to 100
c FOR CASE OF S1 - ON & S2 - ON
c ----------------------------
        y(2)=0.0d0
        y(4)=0.0d0
 200    rka(1)=dt*(vs/l1)
        rka(3)=dt*(vs/l2)
        rka(5)=-dt*(y(6)/lo+y(5)*r/lo)
        rka(6)=dt*(y(5)/co)
        yy(1)=y(1)+rka(1)/2.0d0
        yy(3)=y(3)+rka(3)/2.0d0
        yy(5)=y(5)+rka(5)/2.0d0
        yy(6)=y(6)+rka(6)/2.0d0
        rkb(1)=dt*(vs/l1)
        rkb(3)=dt*(vs/l2)
        rkb(5)=-dt*(yy(6)/lo+yy(5)*r/lo)
        rkb(6)=dt*(yy(5)/co)
        yyy(1)=yy(1)+rkb(1)/2.0d0
        yyy(3)=yy(3)+rkb(3)/2.0d0
        yyy(5)=yy(5)+rkb(5)/2.0d0
        yyy(6)=yy(6)+rkb(6)/2.0d0
        rkc(1)=dt*(vs/l1)
        rkc(3)=dt*(vs/l2)
        rkc(5)=-dt*(yyy(6)/lo+yyy(5)*r/lo)
        rkc(6)=dt*(yyy(5)/co)
        yyyy(1)=yyy(1)+rkc(1)
        yyyy(3)=yyy(3)+rkc(3)
        yyyy(5)=yyy(5)+rkc(5)
        yyyy(6)=yyy(6)+rkc(6)
        rkd(1)=dt*(vs/l1)
        rkd(3)=dt*(vs/l2)
        rkd(5)=-dt*(yyyy(6)/lo+yyyy(5)*r/lo)
        rkd(6)=dt*(yyyy(5)/co)
        y(1)=y(1)+(1.0d0/6.0d0)*(rka(1)+2.0d0*rkb(1)+2.0d0*rkc(1)+rkd(1)
        y(3)=y(3)+(1.0d0/6.0d0)*(rka(3)+2.0d0*rkb(3)+2.0d0*rkc(3)+rkd(3)
        y(5)=y(5)+(1.0d0/6.0d0)*(rka(5)+2.0d0*rkb(5)+2.0d0*rkc(5)+rkd(5)
        y(6)=y(6)+(1.0d0/6.0d0)*(rka(6)+2.0d0*rkb(6)+2.0d0*rkc(6)+rkd(6)
        t=t+dt
```

```
      ss1=y(1)-y(5)
      ss2=y(5)+y(3)
      if(m.ne.1) go to 2
      if(n.eq.10) then
      n=0
      write(21,*)t,y(1)
      write(22,*)t,y(2)
      write(23,*)t,y(3)
      write(24,*)t,y(4)
      write(25,*)t,y(5)
      write(26,*)t,y(6)
      write(27,*)t,ss1
      write(28,*)t,ss2
      endif
      n=n+1
 2    if(t.lt.(t1+t2)) go to 200
c FOR CASE OF  S1 _ ON & S2 - OFF
c-------------------------------
 300  y(2)=0.0d0
      rka(1)=dt*(vs/l1)
      rka(3)=dt*(-y(4)/l2+vs/l2)
      rka(4)=dt*(y(3)+y(5))/c2
      rka(5)=-dt*((y(4)+y(6))/lo+y(5)*r/lo)
      rka(6)=dt*y(5)/co
      do 9 i=1,6
 9    yy(i)=y(i)+rka(i)/2.0d0
      rkb(1)=dt*(vs/l1)
      rkb(3)=dt*(-yy(4)/l2+vs/l2)
      rkb(4)=dt*(yy(3)+yy(5))/c2
      rkb(5)=-dt*((yy(4)+yy(6))/lo+yy(5)*r/lo)
      rkb(6)=dt*yy(5)/co
      do 66 i=1,6
 66   yyy(i)=yy(i)+rkb(i)/2.0d0
      rkc(1)=dt*(vs/l1)
      rkc(3)=dt*(-yyy(4)/l2+vs/l2)
      rkc(4)=dt*(yyy(3)+yyy(5))/c2
      rkc(5)=-dt*((yyy(4)+yyy(6))/lo+yyy(5)*r/lo)
      rkc(6)=dt*yyy(5)/co
      do 77 i=1,6
 77   yyyy(i)=yyy(i)+rkc(i)
      rkd(1)=dt*(vs/l1)
      rkd(3)=dt*(-yyyy(4)/l2+vs/l2)
      rkd(4)=dt*(yyyy(3)+yyyy(5))/c2
      rkd(5)=-dt*((yyyy(4)+yyyy(6))/lo+yyyy(5)*r/lo)
      rkd(6)=dt*yyyy(5)/co
      do 88 i=1,6
 88   y(i)=y(i)+(1.0d0/6.0d0)*(rka(i)+2.0d0*rkb(i)+2.0d0*rkc(i)+rkd(i)
      t=t+dt
      if(y(4).lt.0.0d0)then
      y(4)=0.0d0
      endif
      ss1=y(1)-y(5)
      ss2=y(3)+y(5)
      y(2)=0.0d0
      if(m.ne.1) go to 33
      if(n.eq.10) then
      n=0
      write(21,*)t,y(1)
      write(22,*)t,y(2)
      write(23,*)t,y(3)
      write(24,*)t,y(4)
      write(25,*)t,y(5)
      write(26,*)t,y(6)
      write(27,*)t,ss1
      write(28,*)t,ss2
      endif
```

```
      n=n+1
 33   if(t.lt.(t1+t2+t3)) go to 300
c FOR CASE OF S1 - ON & S2 - ON
c ------------------------------
      y(2)=0.0d0
      y(4)=0.0d0
400   rka(1)=dt*(vs/l1)
      rka(3)=dt*(vs/l2)
      rka(5)=-dt*(y(6)/lo+y(5)*r/lo)
      rka(6)=dt*(y(5)/co)
      yy(1)=y(1)+rka(1)/2.0d0
      yy(3)=y(3)+rka(3)/2.0d0
      yy(5)=y(5)+rka(5)/2.0d0
      yy(6)=y(6)+rka(6)/2.0d0
      rkb(1)=dt*(vs/l1)
      rkb(3)=dt*(vs/l2)
      rkb(5)=-dt*(yy(6)/lo+yy(5)*r/lo)
      rkb(6)=dt*(yy(5)/co)
      yyy(1)=yy(1)+rkb(1)/2.0d0
      yyy(3)=yy(3)+rkb(3)/2.0d0
      yyy(5)=yy(5)+rkb(5)/2.0d0
      yyy(6)=yy(6)+rkb(6)/2.0d0
      rkc(1)=dt*(vs/l1)
      rkc(3)=dt*(vs/l2)
      rkc(5)=-dt*(yyy(6)/lo+yyy(5)*r/lo)
      rkc(6)=dt*(yyy(5)/co)
      yyyy(1)=yyy(1)+rkc(1)
      yyyy(3)=yyy(3)+rkc(3)
      yyyy(5)=yyy(5)+rkc(5)
      yyyy(6)=yyy(6)+rkc(6)
      rkd(1)=dt*(vs/l1)
      rkd(3)=dt*(vs/l2)
      rkd(5)=-dt*(yyyy(6)/lo+yyyy(5)*r/lo)
      rkd(6)=dt*(yyyy(5)/co)
      y(1)=y(1)+(1.0d0/6.0d0)*(rka(1)+2.0d0*rkb(1)+2.0d0*rkc(1)+rkd(1))
      y(3)=y(3)+(1.0d0/6.0d0)*(rka(3)+2.0d0*rkb(3)+2.0d0*rkc(3)+rkd(3))
      y(5)=y(5)+(1.0d0/6.0d0)*(rka(5)+2.0d0*rkb(5)+2.0d0*rkc(5)+rkd(5))
      y(6)=y(6)+(1.0d0/6.0d0)*(rka(6)+2.0d0*rkb(6)+2.0d0*rkc(6)+rkd(6))
      t=t+dt
      ss1=y(1)-y(5)
      ss2=y(3)+y(5)
      if(m.ne.1) go to 22
      if(n.eq.10) then
      n=0
      write(21,*)t,y(1)
      write(22,*)t,y(2)
      write(23,*)t,y(3)
      write(24,*)t,y(4)
      write(25,*)t,y(5)
      write(26,*)t,y(6)
      write(27,*)t,ss1
      write(28,*)t,ss2
      endif
      n=n+1
22    if(t.le.(t1+t2+t3+t4)) go to 400
      do 12 i=1,6
      yt2(i)=y(i)
      err(i)=abs(yt2(i)-yt1(i))
12    continue
      if(err(1) .gt. 0.1d-8) go to 1000
      if(err(2) .gt. 0.1d-8) go to 1000
      if(err(3) .gt. 0.1d-8) go to 1000
      if(err(4) .gt. 0.1d-8) go to 1000
      if(err(5) .gt. 0.1d-8) go to 1000
      if(err(6) .gt. 0.1d-8) go to 1000
```

```
if(m.lt.2) go to 1000
close(21)
close(22)
close(23)
close(24)
close(25)
close(26)
close(27)
close(28)
stop
end
```